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EXACT TRAVELLING WAVE SOLUTIONS FOR MANAKOV SYSTEM BY USING FIRST INTEGRAL METHOD

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Abstract

The aim of this study is to find exact solutions for Manakov system by using the first integral method. This method depends on reciprocal algebra theory in getting an exact solution for nonlinear equations. By means of this method, some new forms for the exact solutions of Manakov system are formally obtained. Results are shown that the proposed method is effective and general.

INTRODUCTION

Recently, most of researchers go on the way of finding exact solutions for nonlinear differential equations. They are very important in studying nonlinear phenomena that are described by differential equations because they have a major role in physics and applied sciences such as fluid dynamics, solid state physics, mechanics, biology and mathematical finance. Some researchers used different methods for solving these equations and finding exact solutions for them such as tanh-sech method[14, 16], extended tanh method[1, 7, 18], the generalized hyperbolic function method[8], sine-cosine method[17], the transformed rational function method[13] and F-expansion method[9]. In such methods, they use to transform isolated wave solutions so as to change nonlinear partial differential equation to ordinary differential equation easily solved by these methods. The first integral method that was suggested by Feng[6] is one of the methods are used to find the exact solutions for different nonlinear partial differential equations as[3, 4, 15].

The main aim of this paper is to apply the first integral method to solve the following Manakov system of nonlinear coupled one-dimensional partial differential equations;

$$iu_t + u_{xx} + (|u|^2 + |v|^2)u = 0\,, \qquad (1a)$$

$$iv_t + v_{xx} + (|u|^2 + |v|^2)v = 0 \qquad (1b)$$

This system and close form to it appearing in many studies such as [10, 11, 12] and the references therein. An important goal of the present work is to show that the efficiency and ability of the first integral method for finding new forms for the exact solutions of nonlinear coupled one-dimensional partial differential equations of Manakov system. The results show that the first integral method is an efficient technique for the solutions of Manakov system.

The first integral method

Let us consider the nonlinear partial differential equation in the form:

$$F(u, u_x, u_t, u_{xx}, \dots \dots ..) = 0 \qquad (2)$$

where $u(x,t)$ represents the exact solution for (2). We use the following transformation:

$$u(x,t) = f(\xi) \qquad (3)$$

where $\xi = x - ct$ and c is a constant. This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = -c\frac{d}{d\xi}(.),\ \frac{\partial}{\partial x}(.) = \frac{d}{d\xi}(.),\ \frac{\partial^2}{\partial x^2}(.) = \frac{d^2}{d\xi^2}(.), \qquad (4)$$

Using (4) to transfer the nonlinear partial differential equation (2) to nonlinear ordinary differential equation in the form

$$Q\ (f(\xi), f'(\xi), f''(\xi) \ldots \ldots .) = 0 \qquad (5)$$

Next, we define new independent variables:

$$X\ (\xi)\ = f\ (\xi),\ Y\ (\xi)\ = f_\xi\ (\xi) \qquad (6)$$

This leads to a system of nonlinear ordinary differential equations:

$$\begin{cases} X_\xi(\xi) = Y(\xi), \\ Y_\xi(\xi) = F\big(X(\xi), Y(\xi)\big). \end{cases} \qquad (7)$$

From the qualitative theory of ordinary differential equations[2], if one can find two first integrals to system (7) under the same conditions, then the exact solutions to system (7) can be solved directly. However, it is difficult to realize this even for a single first integral, because for a given plane autonomous system, there is no general theory telling us how to find its first integrals in a systematic way. We use the division theorem to obtain one first integral to equation (7) which reduces equation (5) to a first order integrable ordinary differential equation. An exact solution to equation (1) is then obtained by solving this equation. For convenience, first let us recall the division theorem for two variables in the complex domain.

Division Theorem[5]: Suppose that $P(w,z)$ and $Q(w,z)$ are polynomials of two variables w and z in complex domain $C[w,z]$ and $P(w,z)$ is irreducible in $C[w,z]$. If $Q(w,z)$ vanishes at all zero points of $P(w,z)$, then there exists a polynomial $G(w,z)$ in $C[w,z]$ such that

$$Q(w,z) = P(w,z)G(w,z)$$

3. Exact solution for Manakov system

Assume that equation (1) has traveling wave solutions in the form:

$$u(x,t) = f(\xi)\ , \qquad v(x,t) = g(\xi) \qquad (8)$$

where $\xi = x - ct$ and c is a constant. By using (8) equations (1a) and (1b) become:

$$-icf'(\xi) + f''(\xi) + \big(f^2(\xi) + g^2(\xi)\big)f(\xi) = 0 \qquad (9a)$$

$$-icf'(\xi) + f''(\xi) + \big(f^2(\xi) + g^2(\xi)\big)f(\xi) = 0 \qquad (9a)$$

Suppose $\alpha = -(f^2(\xi) + g^2(\xi))$ this implies that:

$$g(\xi) = \mp i\sqrt{f^2(\xi) + \alpha} \qquad (10)$$

Then equation (9a) becomes:

$$f''(\xi) = icf'(\xi) + \alpha f(\xi) \quad (11)$$

Using (6) in (11) we obtain

$$X_\xi(\xi) = Y(\xi) \quad (12a)$$

$$Y'(\xi) = icY(\xi) + \alpha X(\xi) \quad (12b)$$

Now, we are applying the division theorem to seek the first integral method to equation (1). Suppose that $X(\xi)$ and $Y(\xi)$ are the nontrivial solutions to equations (12a) and (12b), and $q(X,Y) = \sum_{i=0}^{m} a_i(X)\, Y^i = 0$ is an irreducible polynomial in the complex domain $C[X,Y]$ such that:

$$q[X(\xi), Y(\xi)] = \sum_{i=0}^{m} a_i(X) Y^i = 0 \quad (13)$$

where $a_i(x)$ $(i = 0,1,\dots\dots,m)$ are polynomials of X and all relatively prime in $C[X,Y]$, $a_m(X) \neq 0$. Equation (13) is also called the first integral to (12). Assuming that m=2, note that $\frac{dq}{d\xi}$ is a polynomial in X and Y, and $q[X(\xi), Y(\xi)] = 0$ implies $\frac{dq}{d\xi}$=0, due to the division theorem, there exists a polynomial $(g(X) + h(X)Y)$ in $C[X,Y]$ such as

$$\frac{dq}{d\xi} = \frac{\partial q}{\partial X}\frac{\partial X}{\partial \xi} + \frac{\partial q}{\partial Y}\frac{\partial Y}{\partial \xi} = (g(X) + h(X)Y) \sum_{i=0}^{2} a_i(X)\, Y^i \quad (14)$$

Comparing the coefficients of $Y^i (i = 2,1,0)$ on both sides of equation (14) we get:

$$a'_2(X) = a_2(X)h(X) \quad (15a)$$

$$a'_1(X) = a_2(X)g(X) + a_1(X)h(X) - 2ica_1(X) \quad (15b)$$

$$a'_0(X) = a_1(X)g(X) + a_0(X)h(X) - ica_1(X) - 2\alpha a_2(X)X$$

(15c)

$$\alpha a_1(X)X = a_0(X)g(X) \quad (15d)$$

Since $a_2(X)$ is polynomial of X then, from (15a), we conclude that $a_2(X)$ is constant and $h(X) = 0$. To simplify, we take $a_2(X)$ =1 and balancing the degrees of $g(X)$, a_1(X) and $a_0(X)$ we conclude that deg $g(X)$ = 0 only. Now we discuss this case:

If deg $g(X) = 0$, suppose that $g(X) = A_1$, then we can find $a_1(X)$ and a_0(X):

$$a_1(x) = (A_1 - 2ic)X - B_0 \quad (16)$$

$$a_0(X) = d + (A_1B_0 - icB_0)X + \frac{{A_1}^2 - 3icA_1 - 2c^2 - 2\alpha}{2} X^2 \quad (17)$$

where A_0 and B_0 are arbitrary integration constants. Substituting $a_0(X)$, and $g(X)$ in (15d) and setting all the coefficients of powers X to be zero. We obtain a system of nonlinear algebraic equations:

$$2\alpha A_1 - 2ic\alpha = \frac{{A_1}^3}{2} - \frac{3ic{A_1}^2}{2} - A_1c^2 \quad (18a)$$

$$\alpha B_0 = {A_1}^2 B_0 - icB_0A_1 \quad (18b)$$

$$A_1 d = 0 \quad (18c)$$

Solving the last algebraic equations, we obtain:

$$A_1 = 0\ , \qquad \alpha = 0 \tag{19a}$$

$$A_1 = ic\ , \quad d = 0, \quad \alpha = 0 \tag{19b}$$

Using the condition (19a) into (13), we obtain:

$$Y(\xi) = icX(\xi) + \frac{\sqrt{B_0{}^2-4d}\ -B_0}{2} \tag{20a}$$

$$Y(\xi) = icX(\xi) - \frac{\sqrt{B_0{}^2-4d}+B_0}{2} \tag{20b}$$

Combining (20a) with (12), we obtain the exact solution to (11) as follows:

$$f(\xi) = e^{ic(\xi+\xi_0)} + \frac{i\sqrt{B_0{}^2-4d}-iB_0}{2c} \tag{21}$$

Substituting (21) in (10) we get:

$$g(\xi) = \mp i(\frac{i\sqrt{B_0{}^2-4d}-iB_0}{2c} + e^{ic(\xi+\xi_0)}) \tag{22}$$

where ξ_0 is an arbitrary constant.Then the exact solutions to equation(1) can be written as:

$$u(x,t) = e^{ic(x-ct+\xi_0)} + \frac{i\sqrt{B_0{}^2-4d}-iB_0}{2c} \tag{23}$$

$$v(x.t) = \mp i(e^{ic(x-ct+\xi_0)} + \frac{i\sqrt{B_0{}^2-4d}-iB_0}{2c}) \tag{24}$$

Similarly, for (20b) the exact solution to (11) is:

$$f(\xi) = e^{ic(\xi+\xi_0)} + \frac{iB_0-i\sqrt{B_0{}^2-4d}}{2c} \tag{25}$$

Substituting (25) in (10) we get:

$$g(\xi) = \mp(ie^{ic(\xi+\xi_0)} - \frac{B_0-\sqrt{B_0{}^2-4d}}{2c}) \tag{26}$$

Then the exact solutions to equation (1) can be written as:

$$u(x,t) = e^{ic(x-ct+\xi_0)} + \frac{iB_0-i\sqrt{B_0{}^2-4d}}{2c} \tag{27}$$

$$v(x,t) = \mp(ie^{ic(x-ct+\xi_0)} - \frac{B_0-\sqrt{B_0{}^2-4d}}{2c} \tag{28}$$

By using the condition (19b) in (13) we get:

$$Y(\xi) = icX(\xi) - B_0 \tag{29}$$

Combining (29) with (14), we obtain the exact solution to (11) as follows:

$$f(\xi) = e^{ic(\xi+\xi_0)} - \frac{iB_0}{c} \tag{30}$$

Substituting (30) in (10) we get:

$$g(\xi) = \mp(\frac{B_0}{c} + ie^{ic(\xi+\xi_0)}) \tag{31}$$

Then the exact solutions to equation (1) can be written as:

$$u(x,t) = e^{ic(x-ct+\xi_0)} - \frac{iB_0}{c} \tag{32}$$

$$v(x,t) = \mp(\frac{B_0}{c} + ie^{ic(x-ct+\xi_0)}) \tag{33}$$

These solutions are all new exact solutions. In additional to all exact solutions were put back into the corresponding systems, by means of Maple software and their satisfactions confirm the validity of the solutions obtained in this paper.

CONCLUSIONS

In this paper, new forms for the exact solutions for Manakov system are gotten by using first integral method. Thus, we deduced that the proposed method can be extended to solve many nonlinear partial differential equations, which are arising in the theory of solitons and other areas

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