Research Article *ISSN: 2319-507X*
Swapnil Band, IJPRET, 2013; Volume 1(8): 150-157 *IJPRET*

INTERNATIONAL JOURNAL OF PURE AND APPLIED RESEARCH IN ENGINEERING AND TECHNOLOGY

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EFFECT OF BOUNDARY CONDITIONS ON ANALYSIS OF CYLINDRICAL SHELLS USING FINITE ELEMENT METHOD

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Accepted Date:

27/02/2013

Publish Date:

01/04/2013

Keywords

Shell,

FEM,

Shape function

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Abstract

Shell structures enjoy the unique position of having extremely high aesthetic value in various architectural designs. Shell has wide application in structural engineering. It is important that the best engineering solution ensues, other things being equal, at the expense of the selection of structural form and by not increasing the strength properties of the structure. Shell structures support applied external forces efficiently by virtue of their geometrical form. For finite element analysis of shell structure, the formulation of the shell is based on the basic concept of Ahmed shell element where the three-dimensional solid element is degenerated with the help of extractions obtained from the consideration that one of the dimensions across the shell thickness is sufficiently small compared to other dimensions. The shell element chosen is eight noded isoparametric element which is mapped on -1 ≤ (ξ, η, Ψ)≤+1 natural coordinate system. Each node is associate with five degree of freedom which is $u, v, w, \theta_x, \theta_y$ .Effect of boundary conditions on behavior of shell has been discussed in this work.

INTRODUCTION

Shell as structural elements occupy a leadership position in engineering and, in particular, in civil, mechanical, architectural, aeronautical, and marine engineering. Examples of shell structures in civil and architectural engineering are large-span roofs, liquid-retaining structures and water tanks, containment shells of nuclear power plants, and concrete arch domes. In mechanical engineering, shell forms are used in piping systems, turbine disks, and pressure vessels. Aircrafts, missiles, rockets, ships and submarines are examples of the use of shells in aeronautical and marine engineering.

Another application of shell engineering is in the field of biomechanics, shells are found in various biological forms, such as eye and skull, and plant and animal shapes. This is only a small list of shell forms in engineering and nature. The wide application of shell structures in engineering conditioned by their following advantages:

- Efficiency of load-carrying behavior.
- High degree of reserved strength and structural integrity.
- High strength to weight ratio.
- Very high stiffness.
- Containment of space.

It is important that the best engineering solution ensues, other things being equal, at the expense of the selection of structural form and by not increasing the strength properties of the structure, e.g., by increasing its cross section. Note that the latter approach is easier. Shell structures support applied external forces efficiently by virtue of their geometrical form, i.e., spatial curvatures; as a result, shells are much stronger and stiffer than other structural forms.

The term shell is applied to bodies bounded by two curved surfaces, where the distance between the surfaces is small in comparison with other body dimensions. The locus of points that lie at equal distances from these two curved surfaces defines the middle surface of the shell. The length of the segment, which is perpendicular to the curved surfaces, is called the thickness of the shell and is denoted by h. The geometry of a shell is entirely defined by specifying the form of the middle surface and thickness of the shell at each point.

CLASSIFICATION OF SHELLS

Shells are classified in to various categories depending on various parameters. Some of classifications are given bellows:

Based on geometric form:

- **Surfaces of revolution**

Surface of revolution is generated by rotating a plane curve, called the meridian, about an axis that is not necessarily intersecting the meridian. Circular cylinders, cones, spherical or elliptical domes, hyperboloids of revolution, and toroids are some examples of surfaces of revolution.

Surfaces of translation

A surface of translation is defined as the surface generated by keeping a plane curve parallel to its initial plane as we move it along another plane curve. The two planes containing the two curves are at right angles to each other. An elliptic paraboloid is an example of such a type of surfaces.

Based on shell curvature:

Singly curved shells

- These shells have a zero Gaussian curvature. Some shells of revolution (circular cylinders, cones), shells of translation, or ruled surfaces (circular or noncircular cylinders and cones) are examples of singly curved shells.

Doubly curved shells of positive Gaussian curvature

Some shells of revolution (circular domes, ellipsoids and paraboloids of revolution) and shells of translation and ruled surfaces (elliptic paraboloids, paraboloids of revolution) can be assigned to this category of surfaces.

Doubly curved shells of negative Gaussian curvature

This category of surfaces consists of some shells of revolution (hyperboloids of revolution of one sheet) and shells of translation or ruled surfaces (paraboloids, conoids, hyperboloids of revolution of one sheet).

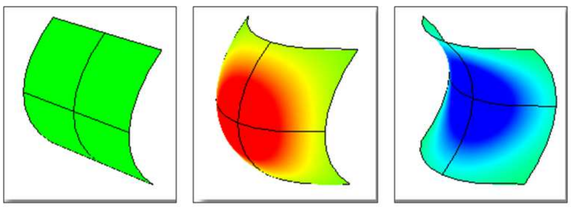

Figure 1: Singly and doubly curved shells

FORMULATION AND MODELING OF SHELL

Formulation of the shell element is based on the basic concept of Ahmed shell where the three-dimensional solid element is degenerated with the help of certain extractions obtained from the consideration that one of the dimensions across the shell thickness is sufficiently small compared to other dimensions (Fig. 2). The element geometry can be nicely represented by the natural coordinate system (ξ, η, ψ) where the curvilinear coordinates (ξ, η) are along the shell mid-surface while z is linear coordinate in the thickness direction. The element is mapped in the region of -1 ≤ (ξ,η,Ψ) ≤ +1

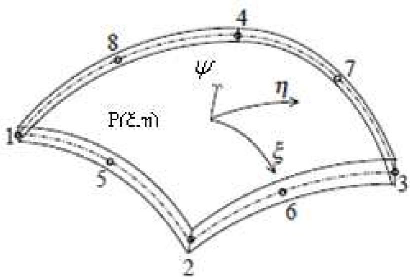

Figure 2: Eight-noded quadrilateral degenerated shell element.

Element displacement matrix

Figure 3 shows an isoparametric eight noded Serendipity element .The nodal displacement vector of an element is given as $\{\delta_e\}_{40x1} = \{u_1 \; v_1 \; w_1 \; \theta_{x1} \; \theta_{y1} \cdots\cdots\cdots \theta_{y8}\}^T$

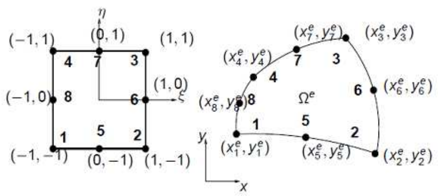

Figure 3: Eight noded Serendipity Element

Displacement of point P (ξ,η) within the element is given in eq.2 which is as follows

$$u = \sum_{i=1}^{8} Ni \cdot ui + \sum_{i=1}^{8} Ni \cdot ui*,$$
$$v = \sum_{i=1}^{8} Ni \cdot vi + \sum_{i=1}^{8} Ni \cdot vi*,$$
$$w = \sum_{i=1}^{8} Ni \cdot wi + \sum_{i=1}^{8} Ni \cdot wi*$$

(2)

Where N_i are the quadratic Serendipity shape functions in (ξ-η), u, v, w are displacements referred to local axis and $u*, v*, w*$ are displacements referred to global axis. Since the shell has very less thickness as compared to other two dimensions, modifications are done to reduce 3D stress-strain relations to 2D relations. Thickness does not change according to deflection. Thus, the final displacements are given in the eq.3.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{4} Ni \begin{Bmatrix} ui \\ vi \\ wi \end{Bmatrix} - \sum_{i=1}^{4} Ni \frac{\psi h}{2} \begin{bmatrix} l1i & l2i \\ m1i & m2i \\ n1i & n2i \end{bmatrix} \begin{Bmatrix} \theta xi \\ \theta yi \end{Bmatrix} = [N_D]\{\delta\}$$

(3)

Geometry of point i.e. co-ordinates (x,y) of point P(ξ,η) is given in eq.4.

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum N_i \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} + \sum N_i \frac{\psi h_i}{2} \begin{Bmatrix} l_{3i} \\ m_{3i} \\ n_{3i} \end{Bmatrix}$$

(4)

where N_i are the quadratic Serendipity shape functions in (ξ-η), h_i is the thickness at the nodal points, {Xi} are Cartesian coordinates at mid-surface nodal points and $\{v_{3i}\}$ is the nodal vectors along the thickness direction. The displacement field may be defined in terms of three displacements components (u_i, v_i, and w_i) and two rotational components (ϑ_{xi} and ϑ_{yi}) at the mid-surface node.

Strain displacement relations

The relations between strain and displacement can be represented by eq.5

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sum_{i=1}^{8} Ni \cdot ui$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y} \sum_{i=1}^{8} Ni \cdot vi$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} = \frac{\partial}{\partial z} \sum_{i=1}^{8} Ni \cdot wi$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial}{\partial y} \sum_{i=1}^{8} Ni \cdot ui + \frac{\partial}{\partial x} \sum_{i=1}^{8} Ni \cdot vi$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \theta x = \frac{\partial}{\partial x} \sum_{i=1}^{8} Ni \cdot wi + \frac{\partial}{\partial z} \sum_{i=1}^{8} Ni \cdot \theta x$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} - \theta y = \frac{\partial}{\partial y} \sum_{i=1}^{8} Ni \cdot wi - \frac{\partial}{\partial z} \sum_{i=1}^{8} Ni \cdot \theta y$$

$$\{\varepsilon\} = [B_1 \quad B_2 \quad B_3 \quad B_4 \quad B_5 \quad B_6 \quad B_7 \quad B_8]\{\delta_e\}$$

$$\{\varepsilon\}_{6\times 1} = [B]_{6\times 40} \{\delta e\}_{40\times 1}$$

Stresses within element

The stresses are related to strains with the help of elasticity matrix [D] as given in eq.6.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \lambda+2G & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2G & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

$$\{\sigma\}_{6\times 1} = [D]_{6\times 6} \{\varepsilon\}_{6\times 1} \qquad (6)$$

Element stiffness matrix of shell

The strain energy of an element is expressed in eq.7.

$$U = \frac{1}{2} \int_V \{\varepsilon\}^T \{\sigma\}\, dV$$

$$= \frac{1}{2} \int_V \{\delta e\}^T [B]^T [D][B]\{\delta e\} \,|\, J \,|\, \partial\xi \partial\eta \partial\psi$$

The stiffness matrix from the above equation can be derived as

$$[Se]_{40\times 40} = \sum_{\xi=-1}^{1} \sum_{\eta=-1}^{1} \sum_{\psi=-1}^{1} [B]^T [D][B] \,|\, J \,|\, \partial\xi \partial\eta \partial\psi$$

(8)

The Jacobian matrix for 3D structure is of order 3x3 which is given below

$$[J]_{3\times 3} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \psi} & \frac{\partial y}{\partial \psi} & \frac{\partial z}{\partial \psi} \end{bmatrix}$$

MODELING OF SHELL

SAP2000(Structural analysis program) is an integrated finite element analysis program for various structures including static and dynamic analysis, supported by powerful analytical capabilities, representing the latest research in numerical techniques and solution algorithms.

NUMERICAL EXAMPLE

In this section of numerical example, study of effect of boundary condition on the behavior of shell has been discussed. Figure 4 shows a cylindrical shell exposed to a concentrated load *P*. This shell structure has four edges out of which two are straight and two are curved. This shell structure has been analyzed for different boundary conditions. The boundary conditions for the shell are simply supported, fixed and free edge. Figure 5 shows the face1, face2, face3 and face4 and pattern of nomenclature. The load applied is 1 kN at center of shell.

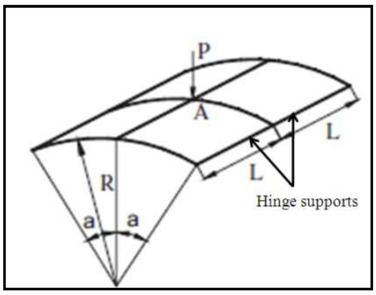

Figure4: Hinged cylindrical roof subjected to a central concentrated point load.

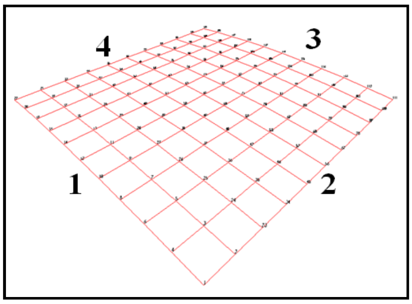

Figure 5: Shell with its boundary faces.

Table 1: Mechanical and geometric properties of shell

Radius of curvature (R)	**2540 mm**
Total length of shell roof (L')	508mm
Roll down angle (a)	5.73 Degree
Modulus of elasticity (E)	3.10275 kN/mm^2
Poisson ratio	0.3
Thickness (t)	25.4 mm

Table 1 shows the boundary conditions and obtained deflections. Here S stand for simply support, C for clamped (fixed) support and F for free edge. As the given shell structure has symmetry about both the axes only quarter part is considered for analysis.

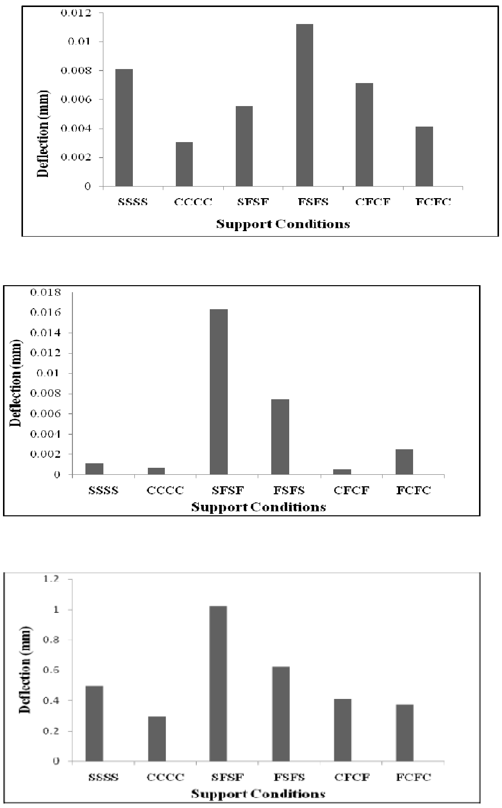

Figure 8: Maximum deflection u, v, w in mm corresponding to different boundary conditions.

CONCLUSION

In the present study, shell formulation is done. At the time of formulation, it is observed that shell behavior is a coupling of axial and bending behavior. Axial displacement causes bending moment and bending displacement causes axial force. Hence analysis has more complexities involved as compared to analysis of plates. Since bending and axial both are involved, predominance of one over other depending upon many parameters like curvatures, span, rise etc. Thus a common simple formulation cannot be applicable to all types of shells. Therefore, here concept most general type of shell element available i.e., 'Ahmed shell' element is used. The behaviour of shell structure for different boundary conditions had been studied. From different combinations of boundary condition it is observed that SFSF i.e., curved edges are simply supported and straight edges are free, is the most flexible configuration amongst all and susceptible to failure.

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