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FINITE ELEMENT ANALYSIS AND DESIGN OF STIFFENED R.C. SLABS

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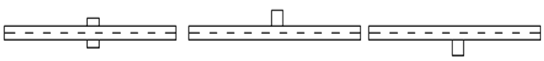
Abstract

A finite element formulation for the analysis of rectangular orthogonally stiffened slab is presented. Results give displacements and stress resultants for the stiffened system considering any geometric, loading and boundary conditions. Using the analysis results design of stiffened system is done. Different cases are carried out considering different orientation and number of stiffener and column members. The stiffened system is descritised into number of three dimensional shell element and stiffener elements. Finite element model consisting of eight noded iso parametric quadrilateral shell element having five degree of freedom coupled with two noded stiffener element having six degrees of freedom are used for the formulation. Design is carried out as per IS 456-2000. Quantitative and cost analysis is carried out for these cases. From the analysis results the most economical case will be decided.

INTRODUCTION

The use of stiffened structural forms in engineering began long ago with the application of steel plates for hulls of ships, bridges and aircraft structures. Seashells, leaves are the examples of naturally stiffened forms. Any structural system is stiffened to achieve maximum structural efficiency defined in terms of stiffness, strength and economy of material. Normally in commercial buildings either in RCC or composite construction, large column free areas or areas with least number of internal columns are desirable. In such situations designers can adopt stiffened slab concept. The designer can achieve more economy by changing the direction and number of stiffeners as well as position of columns in the flooring or roofing system. Therefore an attempt has been made to study the different arrangements and decide which case is more economical.



Slab, one of the important components of a structural system, is often stiffened by ribs or beams to make the system more efficient and economical. The stiffeners may be concentric i.e. its centroidal axis is coincident with the middle surface of the plate or eccentric. The wide use of such composite system in different fields like civil engineering structures, aircraft structures and ship structures had led to the evolution of some simplified methods. Before the use of computers, the analysis of such complex system for all types of conditions was extremely tedious. The high-speed computers revolutionized the methods of structural analysis resulting in development of new analysis techniques like the finite element method. This technique analyses complex systems very efficiently for all sorts of geometric, boundary and loading conditions.

2. SCOPE OF THE PRESENT WORK:

A finite element formulation for the analysis of orthogonal stiffened plates is presented. The displacement and stress fields are given for whole structure for any geometric, loading or boundary conditions. The scenario has changed in recent past with the growing demand among the consulting firms about computer potential. This has led to development of software for analysis and design of various types of structures. In the present work an attempt is made to develop the software by using finite element model for analysis of stiffened plate in SAP-2000(structural analysis program) Software. The analysis output gives displacement, stress resultants and their principal values. The details of the scope of present work are given below: Analysis: Static analysis of stiffened plates using linear elastic theory.

Element:1. Slab is modeled using isoparametric rectangular eight noded plate elements. At each node of plate element five degrees of freedom are considered (u, v, w, θx, and θy).

2. Stiffener is modeled using two noded beam elements. At each node of beam element six degree of freedom are considered (u, v, w, θx, θy and θz).

Geometry of the structure:

1. Shape of structure: Arbitrary.

2. Stiffener configuration (Lay out in plan): Orthogonal parallel stiffeners.

3. Stiffener spacing: Arbitrary

4. Stiffener cross-section: The stiffener may be of different cross section at different positions. However only uniform rectangular cross-section are considered for the numerical study. The bending, shear as well as torsional effects in the stiffener are considered. The material properties of plate and stiffener are considered same.

5. Type of loading: Only transverse loading is considered.

Design: Design of un-stiffened as well as stiffened R.C. slab is carried out as per 456-2000. Numerical problem are solved by considering various cases of large column free areas with different number and orientation of beam and columns. Based on the results of analysis and design for various cases, quantitative and cost analysis is carried out. From the cost analysis results the most economical case has been decided.

3. ELEMENT STIFFNESS MATRIX FOR PLATE:

Consider the 8 noded plate element as shown in following figure-

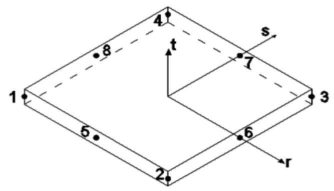


Fig.: Eight Noded Plate Element

The important matrices and relations encountered in the process of finite element formulation are as follows:

Strains = {ε} = [B] {δ}

Stresses = {σ} = [C]$[B]\{\delta\}$ eq.1

4. RECTANGULAR PLATE ELEMENT:

An eight noded rectangular plate element in natural coordinates is shown in figure-

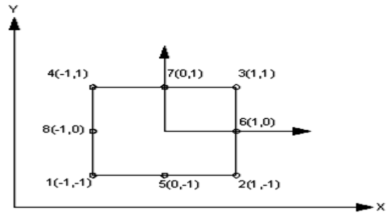


Fig.: Eight Noded Rectangular Element

The variation of displacement u can be expressed by the following polynomial in natural coordinates. $\{u\} = [?][\propto]$

u = ∝1 + ∝2 r + ∝3 s + ∝4 r2 + ∝5 rs + ∝6 s2 + ∝7 r2s +∝8 rs2

eq.2

$[?] = [1 \quad r \quad s \quad r^2 \quad rs \quad s^2 \quad r^2s \quad rs^2]$ eq.3

After substituting and simplifying, the shape functions are obtained as follows:

$N_1 = \frac{1}{4}(1-r)(1-s)(-r-s-1)N_2 = \frac{1}{4}(1+r)(1-s)(r-s-1)N_3 = \frac{1}{4}(1+r)(1+s)(r+s-1)$

$$N_4 = \frac{1}{4}(1-r)(1+s)(-r+s-1)N_5$$

$$= \frac{1}{4}(1+r)(1-r)(1-s)N_6$$

$$= \frac{1}{4}(1+r)(1+r)(1-s)N_7$$

$$= \frac{1}{4}(1+r)(1-r)(1+s)$$

$$N_8 = \frac{1}{4}(1-r)(1+s)(1-s)$$

The shape of element is defined by the eight nodal values as,

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \sum_{i=1}^{8} N_i \begin{Bmatrix} X_i \\ Y_i \\ Z_i \end{Bmatrix}$$

5. DISPLACEMENT FIELD: Using the same shape function, the displacement field within the element is given as

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} N_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix}$$

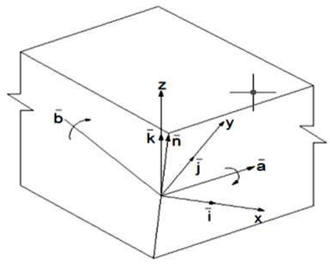


Fig.: Global and Rotational Axis

The relative displacements at i in the global direction are given by,

$[u_i^* \; v_i^* \; w_i^*] = -\frac{1}{2} \quad _i \quad _{xi}[b_1 \; b_2 \; b_3]i + \frac{1}{2} \quad _i \quad _{yi}[a_1 \; a_2 \; a_3]i$ The jacobian matrix can be computed from the following equation:

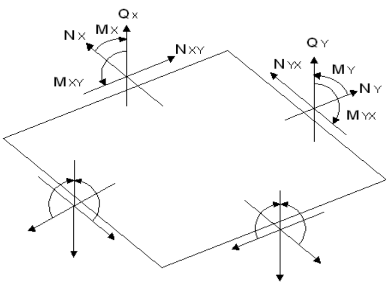
$$[J] = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t} \end{bmatrix}$$

6. ELEMENT STIFFNESS MATRIX:

The stiffness matrix is given by:

$$[K] = \iiint [B][C]\,[B]\delta v$$

the resultant forces and moments per unit length obtained from the internal stresses,



consider an element as shown in fig.

Fig.: Stress Resultant On Plate Element

The compound stresses in shell can be obtained as follows:

$$_x = \frac{N_x}{} + \frac{12M_x z}{^3}$$

$$_y = \frac{N_y}{} + \frac{12M_y z}{^3}$$

$$_{xy} = \frac{N_{xy}}{} + \frac{12M_{xy} z}{^3}$$

The final stiffness matrix for beam element in X-direction is obtained as:

$$k_x = \frac{EI_y}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Similarly considering the degree of freedom of beam as v along y-axis and $_z$ about Z-axis and following the same procedure as above, matrix for element in Y-direction is obtained as under:

$$k_Y = \frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

7. STIFFNESS MATRIX FOR A STIFFENER ELEMENT-ECCENTRIC CONDITION

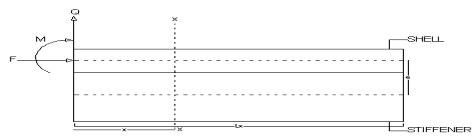


Fig. Stiffened Plate System

bending moment at any section distance 'x' from node of the stiffener is given by:

BM (M1) = M + Fe + Qx

8. STIFFNESS MATRIX FOR ASSEMBLY:

The overall stiffness matrix for the stiffened plate system is obtained as:

$$[K] = [K]_P + [K]_S$$

9. ELEMENT NODAL LOAD VECTOR:

The nodal load vector $\{F_i\}$ at the node I for uniformly distributed load p is given by

$$\{F_i\} = \int_{-1}^{1}\int_{-1}^{1} N^T p|J|dr\, ds$$

10. EXTRAPOLATION OF STRESSES:

The stresses are calculated at the gauss points, which are identified as 1',2',3' and 4'. The gauss point numbering should follow element node numbering The shape function of four nodes are found out using the coordinates of gauss points

$$N'_i = \frac{1}{4}(1 + r')(1 + s')$$

The gauss point stresses are transferred to point stresses with the following relation.

$$_{Nodal} = N'_i \ _{Gauss}$$

11. MODELING OF SHELL

SAP2000(Structural analysis program) is an integrated finite element analysis program for various structures including static and dynamic analysis, supported by powerful analytical capabilities, representing the latest research in numerical techniques and solution algorithms.

12. DESIGN PROCEDURE

Stiffened slab can be idealized as solid slab with a series of stiffeners, which are spaced at regular distances. Loading on the slab is transferred to the beams by one-way or two-way action of the slab. Design procedure of such slabs as per IS 456-2000

13. NUMERICAL EXAMPLE

In this section of numerical example, study of effect of stiffener on the slab has been discussed and also discussed which is most economical.

Fig. shows the stiffener arrangement various cases considered are summarized in Table shows-

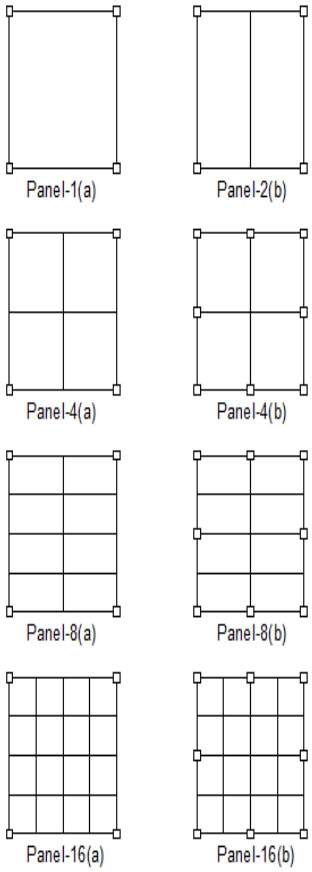


Fig. Various arrangement of stiffener

Table.

No. of panels	No. of stiffener in x-direction	No. of stiffener in y-direction	No. of columns
Panel 1	0	0	4
Panel 2	0	1	4
Panel 4(a)	1	1	4
Panel 4(b)	1	1	8
Panel 8(a)	3	1	4
Panel 8(b)	3	1	8
Panel 16(a)	3	3	4
Panel 16(b)	3	3	8

From the results it is observed that the case of panels 4(b) is most economical case. This

Shows that the stiffened slab becomes economical over un-stiffened slab. The economy depends on the number of stiffeners as well as the column. But this is limited to some extent. If the slab is stiffened beyond certain limit it becomes uneconomical.

CONCLUSIONS

In the present work finite element formulation for the analysis of orthogonally stiffened plate is presented. Software is developed in SAP 2000 (structural analysis program) using finite element model for analysis of stiffened plate. Design of a simple floor slab and R.C.C framed structure is carried out considering different examples by increasing the number of stiffener in the form of beams.

Economical design of R.C.C slab can be obtained by using beams as stiffeners which decreases the deflection in slab panels thus; thickness of the panel can be reduced.

- The finite element method prove to be a very powerful technique for a rigorous analysis of stiffened plate system involving complexities of geometry, boundary conditions and loading conditions to achieve a good degree of accuracy. This method is more suitable to this type of complex problems as compared to other method. It can provide a complete picture of the stress and displacement fields in the structure.
- Supports conditions also plays predominant role in the deflection profile and thus stress-strain pattern also affect.
- Design for each case is done as per IS456. From the design result quantitative and cost analysis is carried out.
- From the results it is seen that stiffened slab becomes economical over unstiffened slab. The economy depends on the orientation and number of stiffener as well as columns.
- From the cost analysis results it seen that from certain point slab becomes uneconomical though it is stiffer. So we can conclude that more fine mesh becomes uneconomical.

REFERENCES

1. Timoshenko S. and Woinowsky-Krieger S.: "Theory of plates and shells", McGraw Hill Book Company.
2. Holmes M. : "Uniform load on a slab or plate stiffened by beams", the structural Engineer, June 1960.
3. Madhujit Mukhopadhyay: "Stiffened plate finite element: An Orthotropic Model for Plate Stress", Journal of Structural Engineering, Madras, Vol.8 No. 1, April 1981.
4. IS 456-2000: Plain and Reinforced Concrete – Code of Practice (Fourth Revision) Bureau Of Indian Standards- June 2000.
5. H.J.: Reinforced Concrete – Vol. 2. Charotar Publishing House (Fifth Edition) 2004