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## ANALYSIS OF SELECTION OF PARAMETERS FOR OPTIMUM THD MEASUREMENT USING DFT

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### Abstract

This paper focuses on analysis of the parameters which affect the Discrete Fourier Transform computation and accuracy. The behaviour of the DFT computation was closely analysed to estimate the error percentage. Also the condition for optimum Total Harmonic Distortion measurement is derived. The results obtained hold true for most methods of DFT computation like Radix2 FFT, Radix4 FFT etc.

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## I. INTRODUCTION

Accurate measurement of THD is crucial for power quality monitoring devices. The THD measurement is done using principles of Fourier transforms. This paper provides designers with guidance for selecting the right parameters in order to perform accurate THD measurements using DFT.

Centralized power systems are now being replaced by distributed generation networks with integration of renewable energy systems into the grid, which make handling variable and large load requirement possible. Most of the integration of renewable power systems to the grid takes place with the aid of power electronics converters. The main purpose of power electronics converters is to integrate the distributed generation to the grid in compliance with power quality standards. However, higher frequency switching of inverters can inject additional harmonics to systems, creating major power quality problems. Therefore it is necessary to monitor the total harmonic distortion at these generation sources to control the connectivity to grid based on THD levels [1]. To accurately estimate the harmonic content it is necessary to implement Discrete Fourier Transform based DSP method in either hardware or

software. The accuracy factor largely depends on the parameters of the Discrete Fourier Transform. Thus, it is important to analyze the parameters and their behaviour under various conditions before selecting the configuration of the DFT.

## II. CONCEPTS

### A. Total Harmonic distortion (THD)

1) *Harmonics*: in terms of voltage or current are the signals with frequency which is an integral multiple of the fundamental frequency. E.g. in India the fundamental frequency in power system is 50Hz. Therefore, harmonics would be 100Hz, 150Hz and 200Hz etc. The harmonic number (h) usually specifies a harmonic component, which is a ratio of its frequency to the fundamental frequency

2) *Total Harmonic Distortion*: THD applies to both current and voltage and is defined as the root mean square (rms) value of harmonics divided by the rms value of the fundamental, and then multiplied by 100% as shown in the following equation:

$$THD = \frac{\sum_{h>1}^{h_{max}} Mh^2}{M1} \times 100 \%$$

In the above equation,  $M_h$  is the amplitude of harmonics with  $h$ , being the harmonics number.  $M_1$  is the amplitude of fundamental frequency component.

## B. Fourier Transforms

The Fourier analysis permits a periodic distorted waveform to be decomposed into an infinite series containing DC component, fundamental component (50/60 Hz for power systems) and its integer multiples called the harmonic components.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

In the equation above,  $c_0 = \frac{a_0}{2}$  is the DC component. The term  $a_n$ ,  $b_n$  correspond to magnitude of the in phase and quadrature phase components at frequency ( $n\omega_0$ ), where  $\omega_0$  is the fundamental frequency in radians given by  $\omega_0 = \frac{2\pi}{T}$  where  $T$  is the time period of the signal. Magnitude at a frequency which is the  $n^{\text{th}}$  harmonic is determined by the formula  $c_n =$

$$\sqrt{a_n^2 + b_n^2}$$

## C. Discrete Fourier Transform (DFT)

A DFT transforms a discrete number of samples in time domain into a frequency spectrum. Using the DFT implies that the finite number of samples being analyzed are taken over one period of an infinitely extended periodic signal<sup>[2]</sup>.

The equation for the Discrete Fourier Transform is:  $F(n) =$

$$\sum_{k=0}^{N-1} x(k) e^{-\frac{j2\pi kn}{N}}$$

Where (1)  $N$  is the number of input samples given for DFT calculation. (2)  $k$  is the index for sample in time domain. (3)  $n$  is the index for sample in frequency domain. (4)  $x(k)$  is the  $k^{\text{th}}$  samples of the input signal  $x$ . (5)  $F(n)$  is the  $n^{\text{th}}$  output sample derived from the DFT calculation in frequency domain.

1) *Points for DFT parameter calculation:*

- $N$  point DFT will take in  $N$  input samples in time domain and produce and a frequency domain output of  $N$  frequencies
- Each input sample is multiplied by sine and cos waves of frequency ( $2\pi kn/N$ )
- The exponential is expressed as  $W_N = e^{-\frac{j2\pi kn}{N}}$  and is called the twiddle factor.
- Euler's formula expresses exponential in terms of the sine and cos waves

given by

$$\text{equation: } e^{-j\frac{2\pi kn}{N}} = \cos\left(\frac{2\pi kn}{N}\right) + j \sin\left(\frac{2\pi kn}{N}\right)$$

- e) The output is complex in nature producing magnitude and phase information
- f) The bin size =  $(f_s/N)$ , the bin size will specify intervals at which the frequencies will be calculated. Here  $f_s$ , is the sampling frequency.
- g) Sampling rate = (samples/cycle)\*(fundamental frequency)
- h) Example: Consider power system, fundamental frequency of signal  $(f) = 50\text{Hz}$ . If signal is sampled at rate 8 samples/cycle, The sampling rate  $(f_s) = (\text{samples/cycle}) * (f) = 400 \text{ samples/sec}$ . Calculation 16 point DFT on the signal, bin size  $= (f_s/N) = (400/16) = 25\text{Hz}$  Therefore, the output will be a range of frequencies [0 25 50 .....375] Hz

## 2) Properties of DFT

- a) *Periodicity*: The twiddle factor  $W$  as per Euler's formula is a complex value composed of sine and cos wave. The sine and cos are periodic with period

$2\pi$ . Thus the DFT also is periodic with period  $(f_s)$ .

- b) *Symmetry*: The twiddle factor  $W$  as per Euler's formula is a complex value composed of sine and cos wave. The sine and cos are symmetric at angle  $2\pi/2$ . Thus the DFT also is symmetric at point  $(f_s/2)$ .
- c) *Interpretation*: Thus the data after range [0 to  $(f_s/2)$ ] at the output is redundant.

## III. ERROR ANALYSIS

### A. Test setup

Making use of the DFT formula mentioned above a mathematical simulation of the DFT was done Excel with the help of macros. The parameters of concern were the Sampling rate  $(S)$  and the number of input samples  $(N)$ . For every combination of Sampling rate  $(S)$  and number of input samples  $(N)$ , the total harmonic distortion (THD) was calculated. The ideal values of amplitude at various frequencies and THD of the test signals were compared with the resultant values obtained by DFT, to determine error percentage.

There were three different test input signals used:

- Sine wave of frequency 50Hz and unit amplitude.
- Sine wave of 50Hz containing harmonics of frequencies up to half sampling frequency( $f_s/2$ )
- Sine wave of 50Hz containing harmonics of frequencies even above ( $f_s/2$ ).

*Amplitude error:* is the percentage of deviation of the obtained amplitude from the actual amplitude.

*THD measurement error:* is the percentage of deviation of the obtained value from the actual THD value present in the signal.

## B. Observation, Results and Analysis

### 1) Amplitude Error Analysis

The results of the simulation for various values of N and S, have been mapped in a tabular form. Each cell of the table contains the amount of amplitude error for each corresponding value of N and S.

#### Case 1:

Table 1 contains the values of fundamental amplitude error for sine wave of 50Hz and unit amplitude .Amplitude error for the fundamental is

independent of N, and it increases as sampling rate increases.

When  $N < S$ , the bin size is greater than fundamental frequency and hence this frequency does not correspond to any point at the output. The DFT output being discrete the intermediate point values cannot be determined. The number of input samples to DFT has to be integral multiples of samples/cycle, which means that DFT can be calculated accurately only when input samples contain at least one cycle of the signal.

**Table 1**

Table	N			
Head	8	16	32	64
S	0.0317	0.0317	0.0317	0.0317
8				
16	-	0.0382	0.0382	0.0383
32	-	-	0.0397	0.0398
64	-	-	-	0.0401

#### Case 2

Table 2 contains the values of fundamental amplitude error for sine wave of 50Hz containing harmonics up to

frequency half the sampling rate. Table 3 shows the amplitude error in measurement of harmonics. It can be observed that the fundamental amplitude and the harmonic amplitude error are independent of N. As S increases the amount of error in fundamental amplitude increases but not significantly while the amount of error in harmonic frequency amplitudes reduces. Higher the harmonic number more is the amount of error in its amplitude measured as compared to lower harmonic.

**Table 2**

Table	N			
Head	8	16	32	64
S	0.0036	0.0033	0.0033	0.0037
8				
16	-	0.0160	0.0164	0.0172
32	-	-	0.0301	0.0304
64	-	-	-	0.0379

**Table 3**

Frequency	S		
	8	16	32
100	0.205	0.0825	0.0011
150	0.5217	0.1734	0.0456
200	-	0.3056	0.0913

250	-	0.5107	0.1514
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*Case 3*

Table 4 contains the fundamental amplitude error for the signal of fundamental 50Hz but containing harmonics even higher than half sampling frequency. Table 5 shows the amplitude error for harmonics measurement. The amplitude error in this case is much higher than the other two signals considered before. As we increase the sampling rate the fundamental amplitude error significantly reduces unlike the previous two cases where fundamental amplitude error had increased. It is also observed that the number of harmonics amplitudes that can be reliably calculated also increase as sampling rate increases.

**Table 4**

Table	N			
Head	8	16	32	64
S	7.9598	7.9560	7.9368	7.8386
8				
16	-	3.9688	3.9664	3.9550
32	-	-	0.0288	0.0292
64	-	-	-	0.0379

**Table 5**

Frequency	N		
	8	16	32
100	3.85	0.0845	0.0825
150	-	0.1767	0.1734
200	8.0581	0.3125	0.3056
250	-	0.5179	0.5107

*Reliability Study of DFT*

Table 6 is a based on the reliability study of DFT. A sample signal is taken consisting of frequencies up to 800Hz and the total harmonic distortion of signal being 5%. The signal was sampled at different rates like 16, 32, 64 samples/cycle and one cycle of sample inputs were given to the DFT calculation module.

**Table 6**

Frequency	S		
	16	32	64
50	1.34	0.039	0.04
100	-	0.22	0.02
150	-	0.35	0.04
200	-	0.48	0.07
250	-	0.62	0.10
300	-	0.77	0.12
350	-	0.93	0.15
400	-	1.10	0.18
450	-	1.38	0.28
500	-	1.60	0.31
550	-	1.88	0.34
600	-	2.22	0.37
650	-	2.66	0.40
700	-	3.33	0.44
750	-	4.66	0.47
800	-	94	0.50

Nyquist criteria states the detectable signals will be those having frequency in a range below half the sampling frequency. But it is observed that the reliable amplitude detections of error percentages less than 1 % occur at even fewer frequencies.

As in case of S=32, sampling frequency (fs)= 32\*50=1600. As, per Nyquist, maximum detectable frequency is 800Hz but in practice the error rate increases for higher frequencies and actual number of frequencies reliably detected are within fs/4.

*Total Harmonics Distortion Measurement Error Analysis*

After analysing the amplitude measurement error for both fundamental and harmonics, a further analysis was done based on the error in measuring THD for the three sample signals for the above cases. The total harmonic distortion is measure for the sample test signals and the value is compared with those obtained from the DFT output. The error percentages have been observed for various conditions.

*Case 1*

Table 7 contains the THD measurement error values for a sine wave of 50 Hz. It is seen that for as S increases the THD measurement error reduces, but THD error is independent of N. The error is significantly high for the condition  $N < S$  where the input samples to the DFT are not even one cycle of the signal.

**Table 7**

Table	N			
Head	8	16	32	64
S	0.0897	0.1200	0.1644	0.2271
8				
16	36.56	0.0592	0.0812	0.1121
32	41.870	34.758	0.0405	0.0559
64	45.959	37.656	34.323	0.0279

**Case 2**

Table 8 corresponds to the signal whose frequencies above the Nyquist frequency ( $f_s/2$ ) have been filtered. From the THD measurement analysis on this signal it is seen that condition  $N \geq S$  is satisfied, the value of N does not affect the THD error, only the increase in S reduces the error in THD measurement.

**Case 3**

Table 9 is for a signal having THD 5% and having frequencies up to 800 Hz. It is observed that as the sampling rate increases the THD measurement error has reduced by a great amount.

**Table 8**

Table	N			
Head	8	16	32	64
S	0.3513	0.3571	0.03571	0.4843
8				
16	27.420	0.4188	0.4217	0.4366
32	49.619	43.745	0.0300	0.2465
64	60.302	63.534	48.44	0.0708

**Table 9**

Table	N			
Head	8	16	32	64
S	98.8	98.8	98.8	98.8
8				
16	-	1.88	1.88	1.88
32	-	-	0.23	0.23
64	-	-	-	0.02



#### IV. CONCLUSION

When using DFT the first dilemma is to select the right parameters. From the over values calculated using the DFT code certain limitations are highlighted.

##### A. DFT parametric facts

From the above analysis we can conclude certain points which can prove to be guide lines for selecting sampling rate as well as the number of point FFT to be calculated.

1. Number of input samples to the DFT has to be greater than or equal to the samples/cycle.
2. The DFT calculate values of amplitude for frequencies which are (N-1) multiples of bin size only, and no intermediate points estimated.
3. The amplitude error is independent of the number of point DFT that is calculated and depends only on sampling rate.
4. The error in amplitude estimation for higher harmonics is larger.
5. For higher sampling rate, higher is the accuracy of harmonic amplitude detection and the THD measurement but the fundamental amplitude accuracy reduces slightly.

6. Reliable DFT amplitude measurement can be done for frequencies up to the one fourth of the sampling frequency.
7. Error analysis is fundamental frequency independent and can be used as guide lines for all cases.

##### B. Choosing the N and S combination for optimum THD measurement

1. Select the maximum frequency to be detected.
2. Let the sampling rate at least be 4 time the amplitude measurement
3. Convert the sampling rate in terms of samples per cycle and choose the sampling rate which give samples/cycle value as power of 2.
4. Number of input points to DFT should choose equal to the number of samples per cycle. This the most accurate combination for THD application where we are intersted in frequencies as integer multiples of fundamental.
5. The maximum sampling rate is limited but the hardware and time constraints.
6. For higher sampling rate we need higher point DFT to be calculated. This may increase computation time and complexity

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