



# INTERNATIONAL JOURNAL OF PURE AND APPLIED RESEARCH IN ENGINEERING AND TECHNOLOGY

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## EFFECTS OF REYNOLDS NUMBER ON LINEAR AND NON- LINEAR JOURNAL TRAJECTORY AT CONSTANT SPEED OF NON RECESSED HYBRID JOURNAL BEARING

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### Accepted Date:

27/02/2013

### Publish Date:

01/04/2013

### Keywords

Dynamic Performance, Hybrid Bearing,  
Non Linear Stability,  
Stable Region,  
Turbulence

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### Abstract

In this paper, the analysis is done for the fluid–flow–field in the clearance space of hybrid journal bearing system using short bearing analysis under laminar transition and turbulent flow conditions. The positive pressure zone is established after deleting sub-ambient pressure around clearance space. Linear and non linear motion trajectories at constant speed are obtained using the fourth order Runge-Kutta method. Journal centre trajectories at constant speed and laminar flow show the expected trend i.e. system is stable below threshold speed curve, unstable above it. But for super laminar flow stable behavior is found at some distance away from the stability curve.

**INTRODUCTION** – In the modern technology, non recessed hybrid journal bearing are generally used in various high speed rotating machineries such as gas turbines, steam turbines, hydraulic turbines, compressor, etc. Capone et al. [1, 2] determined the stiffness and damping coefficients and the stability limit curves for various Reynolds number, for a given value of R/C ratio, of a journal bearing in a non-laminar lubrication regime. Using short bearing approximation for derivation of the Reynolds equation with usual assumptions, Kirk et.al [3] get the rapid solutions of the fluid-film bearing forces and found assumption very good for L/D ratios of 0.5 or less. Linearizing equations of motion using stiffness and damping coefficients, stability analysis for horizontal unloaded journal have been performed, and compared with the analysis of Badgley and Booker's [4] and found that limit on eccentricity at which the journal is completely stable is very nearly the same, i.e. 0.8. In their part II work, Kirk et al. Frene et al. [5] showed that the actual transition region may be sufficiently wide to include many of the practical application. Results

show that the transition zone width increases with eccentricity. To analyze the dynamic characteristics of turbulent journal bearing, Hashimoto et al. [6] had taken short bearing theory and results were compared to the finite bearing theory. At large Sommerfeld number however, the turbulence does not affect the stability. Chinang et al. [7] investigated the linear stability analysis of a rough short bearing lubricated with non-Newtonian fluids. Linear and nonlinear transient analyses in the form of trajectories were investigated for a finite length journal bearing by Chandrawat et al. [8].

In this paper, effect of Reynolds number on Linear and non linear motion trajectories at constant speed are obtained using the fourth order Runge-Kutta method

### FLOW FIELD EQUATIONS

The Reynolds equation which governs the flow of lubricating oil in the clearance space of a hybrid journal bearing using linearized turbulence theory is given by

$$\frac{\partial}{\partial \alpha} \left[ \frac{\bar{R}^3}{\mu R_\alpha} \frac{\partial \bar{g}}{\partial \alpha} \right] + \frac{\partial}{\partial \beta} \left[ \frac{\bar{R}^3}{\mu R_\beta} \frac{\partial \bar{g}}{\partial \beta} \right] = \frac{1}{2} \bar{\Omega} \frac{\partial \bar{R}}{\partial \alpha} + \frac{\partial \bar{R}}{\partial \bar{z}} \quad (1)$$

**Fluid film thickness**

The non dimension fluid-film thickness ( $\bar{h}$ ) for parallel axes case is given by

$$\bar{h} = 1.0 - \bar{X}_j \cos \alpha - \bar{Z}_j \sin \alpha \quad (2)$$

Finally, the non-dimensional Reynolds equation reduces to

$$\frac{\partial}{\partial \alpha} \left[ \frac{\bar{h}^3}{\mu R_\alpha} \frac{\partial \bar{p}}{\partial \alpha} \right] + \frac{\partial}{\partial \beta} \left[ \frac{\bar{h}^3}{\mu R_\beta} \frac{\partial \bar{p}}{\partial \beta} \right] = \frac{1}{2} \Omega (\bar{X}_j \sin \alpha - \bar{Z}_j \cos \alpha) - \bar{X}_j \cos \alpha - \bar{Z}_j \sin \alpha \quad (3)$$

**Short bearing approximation**

If the approximation is made that bearing is infinitely short, such that pressure gradient in the circumferential direction is much smaller than the axial direction, i.e.

$$\frac{\partial \bar{p}}{\partial \alpha} \ll \frac{\partial \bar{p}}{\partial \beta}$$

**Boundary condition**

The boundary conditions pertinent to the problem are given as

- (i)  $\bar{p} = 0$ , at  $\alpha = \alpha_1$
- (ii)  $\bar{p} = 0$ , at  $\alpha = \alpha_2$
- (iii)  $\frac{\partial \bar{p}}{\partial \alpha} = 0$  at  $\alpha = \alpha_2$

$$(iv) \bar{p} = 0, \text{ at } \beta = \pm \frac{L}{D} = \pm \lambda \quad (4)$$

Integrating Eq. (3) and using boundary conditions Eq. (4) the expression for pressure distribution is obtained as

$$\bar{p} = \left( \frac{\mu R_\beta}{\bar{h}^3} \right) \left( \frac{1}{2} f(\alpha) (\beta^2 - \lambda^2) \right) \quad (5)$$

The fluid film pressure is computed using Eq. (5) and to establish positive pressure zone, all negative pressure are made zero.

**Load carrying capacity**

The load carrying capacity of journal bearing is found by integrating the pressure over the positive pressure zone.

$$\bar{F}_x = \frac{F_x C^2}{\mu_r \omega_r R^3} = \int_{-\lambda}^{+\lambda} \bar{p} \cos \alpha \, d\alpha \, d\beta \quad \text{And}$$

$$\bar{F}_z = \frac{F_z C^2}{\mu_r \omega_r R^3} = \int_{-\lambda}^{+\lambda} \bar{p} \sin \alpha \, d\alpha \, d\beta \quad (6)$$

Equation (6) is integrated numerically using gauss-Legendre formula over the positive pressure zone.

**Dynamic case**

The fluid film stiffness coefficients are defined in matrix form as

$$\begin{bmatrix} \overline{K_{xx}} & \overline{K_{xz}} \\ \overline{K_{zx}} & \overline{K_{zz}} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \overline{F_x}}{\partial \overline{x_j}} & \frac{\partial \overline{F_x}}{\partial \overline{z_j}} \\ \frac{\partial \overline{F_z}}{\partial \overline{x_j}} & \frac{\partial \overline{F_z}}{\partial \overline{z_j}} \end{bmatrix} \quad (7)$$

The fluid film damping coefficients are defined in matrix form as

$$\begin{bmatrix} \overline{C_{xx}} & \overline{C_{xz}} \\ \overline{C_{zx}} & \overline{C_{zz}} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \overline{F_x}}{\partial \dot{\overline{x_j}}} & \frac{\partial \overline{F_x}}{\partial \dot{\overline{z_j}}} \\ \frac{\partial \overline{F_z}}{\partial \dot{\overline{x_j}}} & \frac{\partial \overline{F_z}}{\partial \dot{\overline{z_j}}} \end{bmatrix} \quad (8)$$

### Equations of motion

(a) No unbalance

$$\overline{M} \ddot{\overline{x}}_{jm} = \overline{F}_x - \overline{F}_{ox} \quad (9)$$

$$\overline{M} \ddot{\overline{z}}_{jm} = \overline{F}_z - \overline{F}_{oz} \quad (10)$$

### Linearized equations of motion

Within a small neighbourhood of the equilibrium position of the journal centre, the restoring fluid film force components may be assumed to be linear functions of the perturbation co-ordinates ( $x_1, x_2$ ) and the perturbation velocities ( $\dot{x}_1, \dot{x}_2$ ).

Thus,

$$(F_1 + F_{01}) = - \left\{ \sum_{j=1}^2 K_{ij} x_{ij} + \sum_{j=1}^2 C_{ij} \dot{x}_{ij} \right\}$$

Using this, the linearized equations are written as

$$\overline{M} \ddot{\overline{x}}_j + \overline{C}_{11} \dot{\overline{x}}_j + \overline{C}_{12} \dot{\overline{z}}_j + \overline{K}_{11} \overline{x}_j + \overline{K}_{12} \overline{z}_j = 0 \quad (11)$$

$$\overline{M} \ddot{\overline{z}}_j + \overline{C}_{21} \dot{\overline{x}}_j + \overline{C}_{22} \dot{\overline{z}}_j + \overline{K}_{21} \overline{x}_j + \overline{K}_{22} \overline{z}_j = 0 \quad (12)$$

### Stability analysis

The stability of the journal bearing system is determined by applying Routh's Criteria to the characteristic equations of free vibration linearized equations of motion given by Eq. (13) and (14)

$$\overline{M} \ddot{\overline{x}}_1 + \overline{C}_{11} \dot{\overline{x}}_1 + \overline{C}_{12} \dot{\overline{x}}_2 + \overline{K}_{11} \overline{x}_1 + \overline{K}_{12} \overline{x}_2 = 0 \quad (13)$$

$$\overline{M} \ddot{\overline{x}}_2 + \overline{C}_{21} \dot{\overline{x}}_1 + \overline{C}_{22} \dot{\overline{x}}_2 + \overline{K}_{21} \overline{x}_1 + \overline{K}_{22} \overline{x}_2 = 0 \quad (14)$$

The characteristic equation is

$$\lambda^4 + A_1 \lambda^3 + A_2 \lambda^2 + A_3 \lambda + A_4 = 0 \quad (15)$$

For stability, necessary condition is

$$(i) A_1, A_2, A_3, A_4 > 0$$

Sufficient conditions are

$$(i) A_1 A_2 - A_3 > 0$$

$$(ii) A_1 A_2 A_3 - A_3^2 - A_1^2 A_4 > 0 \quad (16)$$

The above conditions give the stability margin of the journal bearing system in terms of critical mass ( $M_c$ ) of the journal. The critical mass ( $M_c$ ) from Eq. (17) is given by

$$M_c = \frac{(C_{11}C_{22} - C_{12}C_{21})}{\frac{(C_{11} + C_{22})(K_{11}K_{22} - K_{12}K_{21})}{(K_{11}C_{22} + K_{22}C_{11} - K_{12}C_{21} - K_{21}C_{12})} - \frac{(K_{11}C_{11} + K_{22}C_{22} + K_{12}C_{21} + K_{21}C_{12})}{(C_{11} + C_{22})}} \quad (17)$$

## RESULTS AND DISCUSSION

The analysis and solution algorithm were used to compute the static and dynamic performance characteristics and to obtain transient motion trajectories. The motion trajectories are obtained for constant speed for the plain circular hydrodynamic journal bearing operating in laminar and super laminar flow conditions. These studies are conducted by taking bearing aspect ratio (L/D) 0.25 and 0.5. Transient analysis is particularly done for 0.5 aspect ratio, assuming bearing and journal axes parallel and ratio of nominal clearance to the journal radius 0.001 ( $C/R = 0.001$ ).

### Validation of results

To establish the validity of the analysis, solution algorithms and the computer

program, the static and dynamic performance characteristics obtained from the present short bearing approximation were compared with short bearing results available in literature [1]. In Fig. 1 and Fig. 2, the comparison of static characteristics such as attitude angle and eccentricity [1] are shown to present short bearing analysis and by comparisons some variations in results are found to be inevitable but they compare well. These comparisons adequately establish the authenticity of the analysis, solution algorithms and the computer program used to obtain results.

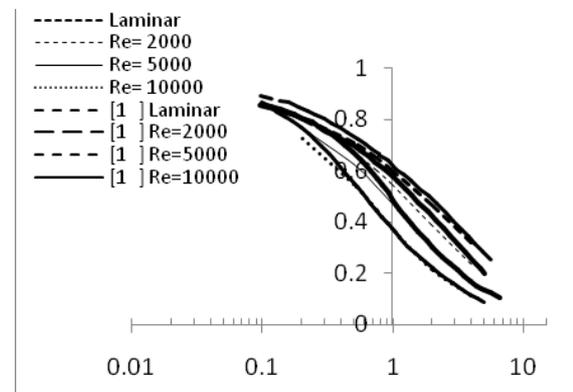


Figure.1 Eccentricity vs Sommerfeld Number for L/D = 0.25 for variation of Reynolds number

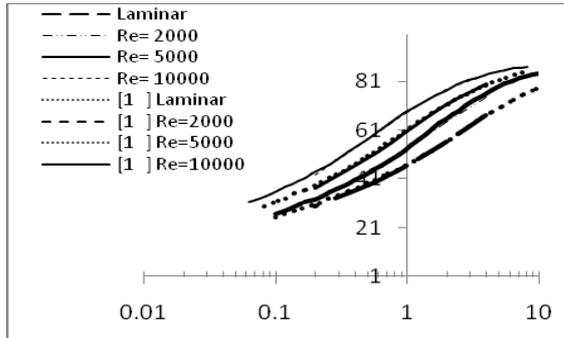


Figure.2 Attitude angle vs Sommerfeld Number for  $L/D = 0.25$  for Variations of Reynolds number

### Transient motion trajectories

In the present work, the linear and nonlinear transient motion trajectories have been obtained for the plain circular non recessed hybrid journal bearing for different Reynolds number using linear and nonlinear equations of motion. Figure 3 shows a typical stability chart in which the critical speed is plotted against Sommerfeld number.

Journal centre trajectories presented in Fig. 4 to Fig. 7 are obtained along with the line OA of Fig. 5, considering points on above and below the threshold speed curve, shown by points 'a' to 'i' on OA line. These trajectories are for plane, circular rigid journal bearing at various value of

Sommerfeld numbers which give non-dimensional constant vertical load  $\bar{W}$ . For all the cases investigated, the values of initial disturbance of positions were 0.005 and disturbance of velocities were 0.0.

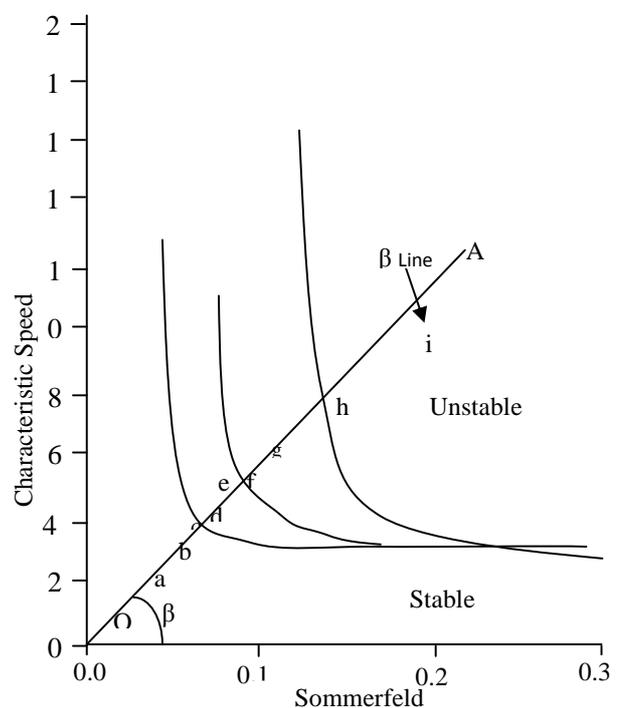


Figure.3 Linerized system stability chart

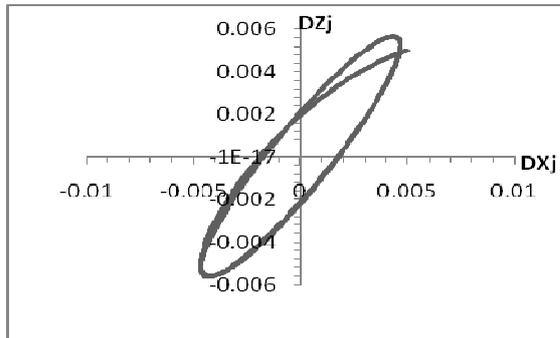


Figure.4 Linear journal centre trajectory at constant speed along line OA ( $Re = 10000$ ,  $L/D = 0.5$ ,  $Sn = 0.0627$  at point 'd')

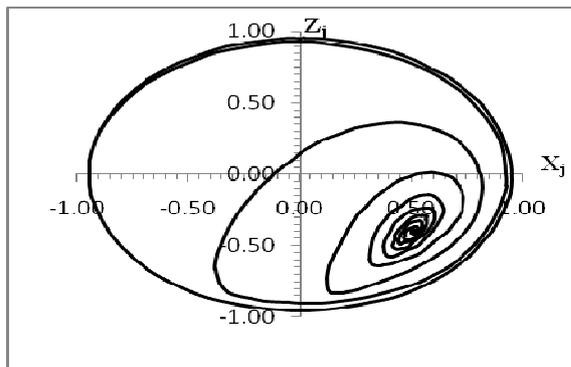


Figure.5 Non-Linear journal centre trajectory at constant speed along line OA ( $Re = 10000$ ,  $L/D = 0.5$ ,  $Sn = 0.0627$  at point 'd')

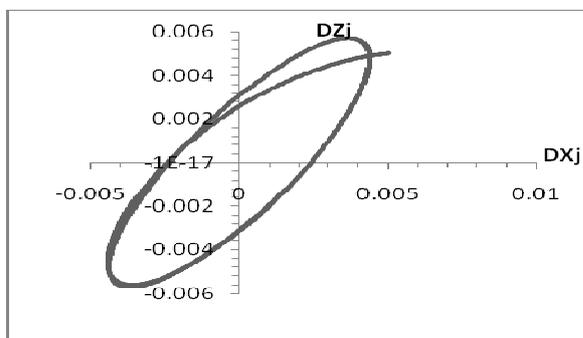


Figure.6 Linear journal centre trajectory at constant speed along line OA ( $Re = 20000$ ,  $L/D = 0.5$ ,  $Sn = 0.05$  at point 'b')

From Fig.4 and Fig.6 it is clear than for linear trajectory, journal is not stable for any value of Reynolds number. Whereas for nonlinear trajectory Fig.5 and Fig.7 shows the stable behavior at constant speed

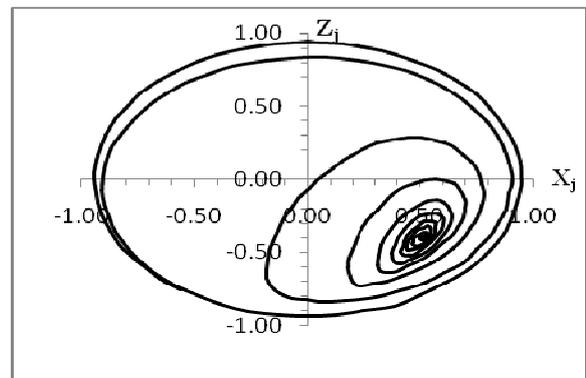


Figure.7 Non-Linear journal centre trajectory at constant speed along line OA ( $Re = 20000$ ,  $L/D = 0.5$ ,  $Sn = 0.05$  at point 'b')

### CONCLUSIONS

It is concluded that for superlaminar flow, stable behavior is found at some distance away from the stability curve and journal makes limit cycle of decreasing magnitude between this stable point and point on the threshold speed curve (when journal mass equals critical mass). Hence the linearized

stability analysis results are not in close agreement with the nonlinear trajectory for superlaminar flows.

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