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MODELLING & SIMULATION OF VIBRATION ENERGY HARVESTING DEVICE

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Abstract

Energy harvesting is a renewable and green power generation and is key technology of future having wide application in NEMS, MEMS, UAV, wireless Sensors, implant medical devices, health monitoring and many more[1]. This paper illustrated the proposed theoretical modeling and simulation, unlike previous models which appear to be mathematical complex, the approach used in this model is simple based on basic piezoelectric, beam theory.

A simple transverse mode type piezoelectric generator model based on Euler–Bernoulli beam theory with the following assumptions is presented: (i) the piezoelectric layer thickness in comparison to the length of the beam is very thin and (ii) the electrical field between the upper surface and lower surface of the piezoelectric layer is uniform. We applied this model to predict the power generated from a bimorph cantilever beam with harmonic oscillations using PVDF as a piezoelectric material. A parametric study is also performed to optimize the energy generation of the system.

I. INTRODUCTION

The rapidly decreasing size, cost, and power consumption of sensors and electronics has opened up the relatively new research field of energy harvesting. The goal, of paper, is to harvest enough ambient energy to power a standalone sensor and/or actuator system. A lot of research has been done in recent years on using ambient vibrations as a power source. Most of this research has been focused on technology-specific solutions. Using piezoelectric principle is one way we can accomplish this [2]. Piezoelectric are the most popular smart materials. Piezoelectric materials have found widespread applications transducers that are able to change electrical energy into mechanical motion or force or vice versa. Roundy et al. [4] presented a approach based on the electrical equivalent circuit but it considered a low vibration and lacks mechanical dynamics of the structure. Egg born [2] developed the analytical models to predict the energy harvesting. Ajitsaria et al. [3] developed modeling and analysis of a bimorph piezoelectric cantilever beam for voltage. In recent years, there have been a considerable number of publications using various models for the electromechanical behavior of piezoelectric energy harvester beams, as can be seen in Anton and Sodano [6]. The models used in the literature range from elementary single-degree-of-freedom (SDOF) models to approximate distributed parameter models as well as analytical distributed parameter solution attempts. Erturk and Inman [7] have recently published a series of papers on energy harvesting using the cantilever model and their work provides a broad coverage of several important modeling aspects.

II. MATHEMATICAL MODELING

The following section describes the development of the piezo models and the analytical estimations of power generation.

i. Piezoelectric Constitution Equation

A model of bimorph cantilever beam is presented. The linear constitutive equations for a piezoelectric material [9] have been employed in terms of the piezoelectric coefficient e_{31}, the permittivity ε_{33}, and the electric field applied across the thickness of the layer E_z.

$D_z = e_{31}\varepsilon_{33} + \varepsilon_{33}E_z$ (1)

The stress σ in z-direction is assumed zero. This occurs when piezoelectric layer thickness in comparison to the length of the beam can be considered very thin.

ii. The Cantilever Beam Model

Figure 1 shows the setup for the cantilever beam model. The PVDF patch is attached to the beam near the clamped edge for maximum strain.

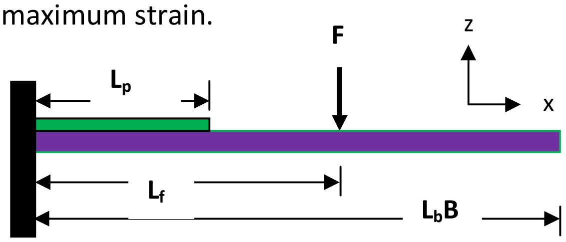

Fig. 1 Cantilever Beam with attached PVDF sensor

The Euler-Bernoulli method is used to model the cantilever beam.

$$\rho a \frac{\partial^2 u(x,t)}{\partial x^2} + E\,I\,\frac{\partial^4 u(x,t)}{\partial x^4} = F(t) \qquad (2)$$

Where *u* is the displacement, *ρ* is density, *a* is area, and *F(t)* is the external force. The boundary conditions are: Considering a harmonic forcing function applied to a single point on the beam, according to Fig. 1, we can write:

$$\frac{\partial^2 u(x,t)}{\partial t^2} + k^2 \frac{\partial^4 u(x,t)}{\partial x^4} = \frac{F_0}{\rho A} \sin(\omega t)\,\delta(x - L_f)$$

(3)

Where ω is the frequency, L_f is the position of the applied force and k is constant.

The frequency will be equal to the beam's first natural frequency because the largest deflection occurs at the first natural frequency. A general solution is given by:

$$w(x,t) = \sum_{i=1}^{3} q_i(t) X_i(x) \qquad (4)$$

Where *qi* is the *i-th* modal coordinate equation of the beam and *Xi* is the *i-th* mode shape of the beam. For consistency, only the first three mode shapes will be used in the summation process described by Eq. 5. Therefore, considering the free undammed vibration,

$$X_i(x)\frac{\partial^2 q_i(t)}{\partial t^2} = -k^2 \frac{\partial^4 X_i(x)}{\partial x^4} q(t)- \qquad (5)$$

Using the standard method of separation of variables in the solution of Eq. 6, we have the equations:

$$-\frac{\frac{\partial^2 q_i(t)}{\partial t^2}}{q(t)} = \frac{k^2 \frac{\partial^4 X_i(x)}{\partial x^4}}{X_i(x)} = \omega_n^2 \qquad (6)$$

Here, we can write the Eq. (7) as:

$$\begin{cases} \frac{\partial^2 q_i(t)}{\partial t^2} + \omega_n{}^2 q_i(t) = 0 \\ \frac{\partial^4 X_i(x)}{\partial x^4} - \frac{\omega_n{}^2}{k^2} X_i(x) = 0 \end{cases} \quad (7)$$

The solution of the 2nd equation in Eq. (7) is:

$$X_i(x) = A_1 \cos(\beta_i x) + A_2 \sin(\beta_i x)$$

$$+A_3 \cosh(\beta_i x) + A_4 \sinh(\beta_i x) \quad (8)$$

Applying the boundary conditions is found the general mode shape equation for a cantilever beam:

$$X_i(x) = \cosh(\beta_i x) - \cos(\beta_i x)$$

$$-\frac{\sinh(\beta_i L_b) - \sin(\beta_i L_b)}{\cosh(\beta_i L_b) + \cos(\beta_i L_b)}(\sinh(\beta_i x) - \sin(\beta_i x)) \quad (9)$$

Where L_b is the beam length.

For the first equation in (7), we have the solution.

$$q_i(t) = \frac{1}{\omega_{di}} e^{-\zeta\omega_{ni}t} \int_0^t F_i(\tau)$$

$$e^{-\zeta\omega_{ni}t} \sin(\omega_{di}(t-\tau))\, d\tau \quad (10)$$

Where *ωd* is the damped natural frequency and ζ i s the damping ratio.

iii. Electromechanical Coupling Modeling

The charge collected on the electrode surface Lu et al [9] can be expressed as the electrical displacement integral on the area of the surface as:

$$Q = b \int_0^{L_b} (e_{31}\varepsilon_x + \epsilon_{33} E_z) dx \quad (11)$$

Where b is the width

Assuming that the voltage *V*, under the uniform electrical field hypotheses, the electric field can be approximately expressed as :

$$E_z = -\frac{V}{t_a} \quad (13)$$

Where t_a the thickness of the piezoelectric layer. Substitution of the equation (13) into (12) leads to:

$$Q = \frac{b t_b e_{31}}{2}[\emptyset(0) - \emptyset(L_p)] - b L_p \epsilon_{33} \frac{V}{t_a} \quad (14)$$

Where ϕ is the slope of deflection of the beam

and t_b is the thickness of the beam.

The amplitude of the current is that of the charge times the frequency that is given as:

$$I = \omega Q \quad (15)$$

For an electrical circuit with pure resistance is expressed as:

$$I = \frac{V}{R} \tag{16}$$

Combining equation (16), (18) and (19) the amplitude of the current can be determined as:

$$I_m = \frac{\omega b t_b e_{31}[\varphi(0) - \varphi(L_b)]}{2\left(1 + bL_b \epsilon_{33}\frac{\omega R}{t_a}\right)} \tag{17}$$

Now the power can be evaluated as:

$$P = {I_m}^2 R \tag{18}$$

$$P = \frac{\omega^2 b^2 {t_b}^2 {e_{31}}^2 [\varphi(0) - \varphi(L_b)]}{4\left(1 + bL_b \epsilon_{33}\frac{\omega R}{t_a}\right)^2} R \tag{19}$$

At x = L_b equation (9) becomes

$$X_i = A \tag{20}$$

$$[\varphi(0) - \varphi(L_b)] = A \tag{21}$$

Where A is the amplitude of vibration

Put (21) in (19)

Power is rewritten as

$$P = \frac{\omega^2 b^2 {t_b}^2 {e_{31}}^2 A}{4\left(1 + bL_b \epsilon_{33}\frac{\omega R}{t_a}\right)^2} R \tag{22}$$

III. NUMERICAL RESULTS AND DISCUSSIONS

The numerical simulations were developed using MATLAB. Table 1 exhibits the beam and the PVDF dimensions and properties used in this work

Table1. Dim and properties of material

Material	Parameter	Value
PVDF	L_p	19.97 mm
	t	0.11 mm
	b	9.80 mm
	E	2.0 GPa
	ρ	1800 kg/m^3
	µ	0.3
	Piezoelectric constant e_{31}	22 x 10^{-12} C/N
	Permitivity ε_{33}	12x ε_0 Dielectric const
		ε_0=1.0 x 10^{-10} F/m
	Relative Permitivity ε	12
Copper	L_b	19.97
	b	9.80

	E	117 GPa
	ρ	8900 kg/m^3
Tungston	Xm	3.99
	Ym	44.99
	Zm	44.98mm
	E	345 GPa
	ρ	19250 kg/m^3

To produce maximum electrical power the structures should be excited at their first natural frequency where they experience the largest deflections [2]. In this study, the beam is excited at the first natural frequency

ω_1= 106.58 Hz. The external system vibrates at excitation acceleration of 0.6g m/s^2 and the mass was placed at the free end of the beam. The PVDF was attached to the beam near the clamped edge and frequency.

Fig 2 shows power vs resistance the maximum power obtained at optimal resistance=645KΩ

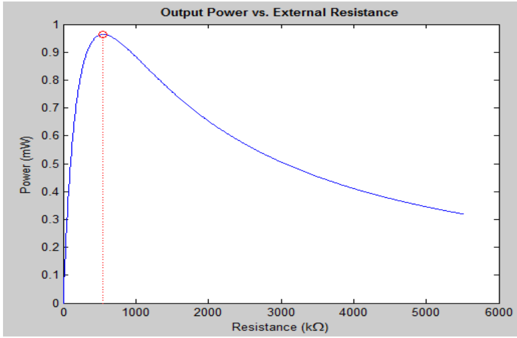

Fig 2. Power as function of the resistance

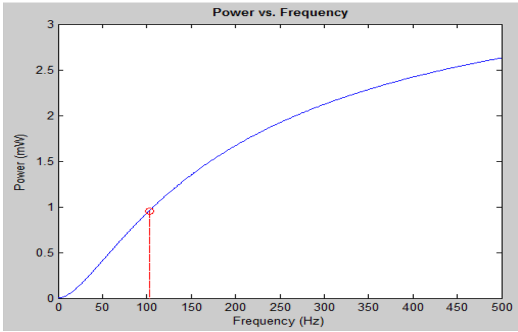

Fig 3. Power as function of the frequency

In the next two figures, it will be considered a variation in the PVDF position and the location of the applied force. Figure 4 shows the location of the applied force varying between 0.01mm and 0.1mm with interval of 0.01m.

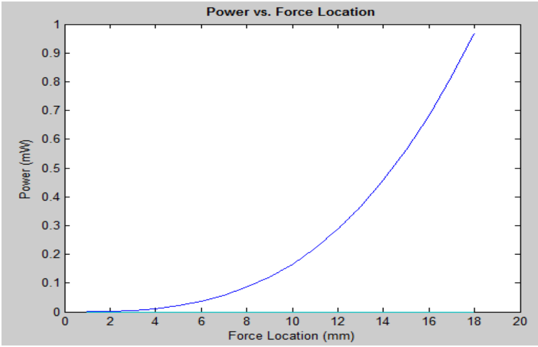

Fig 4. Power vs location of the applied force

The maximum power 1.027 mW was obtained with the force at location for the force is at the free end of the beam. The piezoelectric material location is important to the output power. It must be fixed where there are the largest beam strains. The largest strain occurs at the clamped edge of the cantilever beam .For the next analysis, the PVDF will be sequentially moved along the beam from position *i*=1 to 10, where for each arbitrary position *i*, the PVDF will be localized in $(1-i) \leq L_p \leq iL_b$. This can be seen in figure 5.

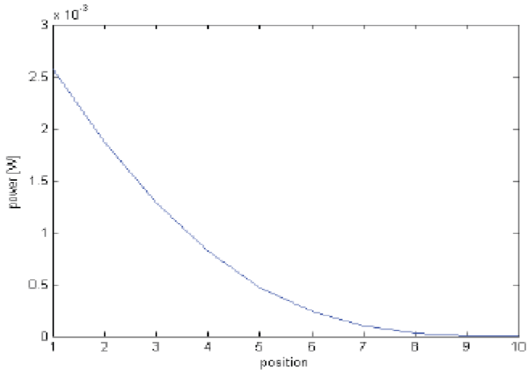

Fig. 5. Power versus position of the PVDF

In the position *i*=1, i.e., $0 \leq x \leq L$, occurs the maximum power. Here it is observed the constant value for the resistance in all positions too. Figures 4 and 5 justify our choice for the force location and PVDF position in figures 2 and 3. For the first natural frequency 106.583 Hz we obtained the maximum power 1.027 mW. Note the direct proportionality between power and resistance. To finish, will be made an investigation about the influence of PVDF length. Initially, the PVDF fixed on the beam's clamped end will have a length, *Lp,* of $L_b/2$ m and will be increased 1mm until it covers the entire length of the beam. The maximum power obtained was 1.027 mW with PVDF length *Lp* = 11mm, approximately when the PVDF covers half the beam.

IV CONCLUSION

This paper gives the simple model of a bimorph cantilever beam for the analysis of piezoelectric power generator application in Wireless Sensor. We have following observation: (i) The Euler-Bernoulli beam theory is assumed, we capture the influence of the natural frequency in power generation for some values of the resistance where the peak value do not vary with the resistance. (ii) The following two assumptions: a) thickness of piezoelectric layer in comparison to the length of the beam is very small and b) the electrical field between the upper surface and lower surface of the piezoelectric layer is uniform.

(iii) A parametric study is performed to optimize the power generation of piezoelectric-beam system. The maximum power was obtained for the applied force in the free-end beam; the piezo located close to clamped end; and piezo length about the half of the beam, but sometimes this is not viable. (iv) it is shown that results obtained from the electrical power calculations indicated that optimal resistance values can be achieved for a particular piezo length and location that will give maximum output power.

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