



# INTERNATIONAL JOURNAL OF PURE AND APPLIED RESEARCH IN ENGINEERING AND TECHNOLOGY

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## STUDY OF METHOD TO DESIGN COMPLEMENTARY ROOT LOCUS

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Accepted Date: 22/09/2013 ; Published Date: 01/10/2013

**Abstract:** In this paper, a method is proposed to draw the Complimentary Root Locus of a given system. It is defined that Complimentary Root Locus can be drawn for the system having Positive feedback (the gain in positive feedback system lies between  $-\infty$  and 0). The stability of this type of system is investigated with the help of both Root Locus and Complimentary Root Locus. Their plots have been presented using suitable example. The Results obtained clearly demonstrate the importance of Complimentary Root Locus. With the help of Complimentary Root Locus, the stability analysis of an oscillator can also be tested. By using problems similar to example presented in the paper, analytical understanding in a classical Control System course can be obtained.

**Keywords:** System Gain, Complimentary Root Locus, Stability, Transfer Function.



PAPER-QR CODE

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How to Cite This Article:

Mahendra Pindel, IJPRET, 2013; Volume 2 (2): 24-29

## INTRODUCTION

The root locus was introduced by WALTER R. EVANS in 1948<sup>1</sup>. This is commonly used tool for graphical design in time domain. This technique can be applied to study the behaviour of roots of algebraic equation with constant coefficient.

Almost all the introductory text books for disciplines about control system in undergraduate engineering courses present the analysis and design of root locus (RL) but don't present information about complementary root locus (CRL)<sup>2</sup>. Some text books present both methods<sup>3-5</sup>.

The RL is designed when transfer function have negative feedback system<sup>6</sup> but the CRL designed when transfer function have positive feedback system<sup>7</sup>. The RL and CRL are used for finding the stability in theoretical manner. The RL and CRL method of analysis is a process of plotting the poles and zeros in s- plane for determining Relative stability of system<sup>8</sup>. In positive feedback system, if the gain of system increases then steady state error decreases or in other words system accuracy is improved. In these methods the designer should know how the closed loop pole moves in s- plane as loop gain varies. For understanding the effect of adding open loop poles and zeros RL is the effective method.

For the system to be stable it is required that all open loop poles should be present in the left half of s- plane<sup>9</sup>.

## MATERIALS AND METHODS

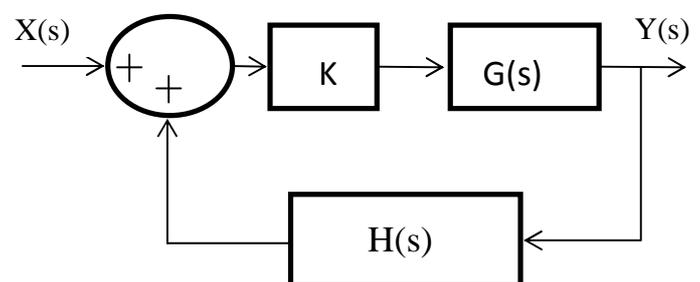


Figure 1. General Transfer Function Block

### A. Definition of CRL

Complimentary Root locus is defined as the locus of open loop poles to open loop zeros obtained when system gain parameter K varies from  $-\infty$  to 0

i.e.  $-\infty < K \leq 0$

Transfer Function Obtained from Figure 1 is

$$\frac{Y(s)}{X(s)} = \frac{K G(s)}{1 - K G(s) H(s)} \quad (1)$$

And Characteristic equation is

$$1 - K G(s) H(s) = 0 \quad (2)$$

### B. Designing Method

CRL can be designed with the help of following theorems

Theorem 1: To find location of poles and zeros when parameter K is varying

$$G(s)H(s) = \frac{a_0s^m + a_1s^{m-1} + \dots + a_m}{b_0s^n + b_1s^{n-1} + \dots + b_n} \quad (3)$$

Where,  $n \geq m$ ,

Using equation (2) and (3), we get that

$$K \left( \frac{a_0s^m + a_1s^{m-1} + \dots + a_m}{b_0s^n + b_1s^{n-1} + \dots + b_n} \right) = 1$$

Remark 1:

At  $K=0$ , it has only open loop poles

$$\frac{a_0s^m + a_1s^{m-1} + \dots + a_m}{b_0s^n + b_1s^{n-1} + \dots + b_n} = \lim_{K \rightarrow 0} \frac{1}{K} \quad (4)$$

$$\text{So, } b_0s^n + b_1s^{n-1} + \dots + b_n = 0 \quad (5)$$

Remark 2:

At  $K = -\infty$ , it has only open loop zeros

$$a_0s^m + a_1s^{m-1} + \dots + a_m = 0 \quad (6)$$

Theorem 2: To find the intersection of Asymptotes

Let  $|K|=1$  in the equation (2),

So that  $|G(s)H(s)| = 1$

It means complimentary root locus is symmetric about real axis.

$$\angle G(s)H(s) = \tan^{-1} \left( \frac{0}{1} \right) = \pm 2\pi q \quad (7)$$

Where, q is an integer (0,1,2,3.....)

For a point to lie on locus, its angle must be an integer multiple of  $2\pi$  about real axis.

Remark 1:

For angle of asymptotes,  $s \rightarrow \infty$

$$G(s)H(s) = \lim_{s \rightarrow \infty} \frac{-k(s + z_1)}{s(s + p_1)(s + p_2)}$$

$$\angle G(s)H(s) = -2\angle s = \pm 2\pi q$$

$$\text{So angle of asymptotes } \angle s = \frac{\pm 2\pi q}{2} \quad (8)$$

Remark 2:

Intersection of asymptotes is always on the real axis, it may or may not be part of locus

$$G(s)H(s) = \frac{-k(s+z_1)(s+z_2)\dots(s+z_m)}{(s+p_1)(s+p_2)\dots(s+p_n)} \quad (9)$$

Where,  $n \geq m$

Putting in equation (9),  $s = \sigma$  because intersection of asymptotes is always real

$$\sigma = \frac{(p_1 + p_2 + \dots + p_n) - (z_1 + z_2 + \dots + z_m)}{n - m}$$

Theorem 3: To find Break-away and Break-in Point

The breakaway point occurs when locus lies between two poles where gain parameter K is having a minimum value. The Break-in point occurs when locus lies between two zeros where gain parameter K is having a

maximum value. Due to the conjugate symmetry of root locus, the break-away point or the break-in point lies either on the real axis or has a complex conjugate pair.

$$K = \frac{b_0s^n + b_1s^{n-1} + \dots + b_m}{a_0s^m + a_1s^{m-1} + \dots + a_n}$$

For a break-away point,

$$\frac{dk}{ds} = 0, \frac{d^2k}{ds^2} \geq 0$$

i.e. condition for minima.

To clarify, the following example is presented.

Example 1: Consider the open-loop transfer function  $G(s)H(s)$  as follows:

$$G(s)H(s) = \frac{K(s + 2)}{2s^3 + 3s^2 + 2s + 2}$$

For this example, one can draw the Root locus and Complimentary root locus to find the stability of the system. For Complimentary root locus, the stability is decided with gain in the range  $-\infty < K < 0$ .

The Root locus is presented in Figure 2, using the conventional methods. Using the Theorems 1,2 and 3, we have drawn the complimentary root locus in s-plane in Figure 3.

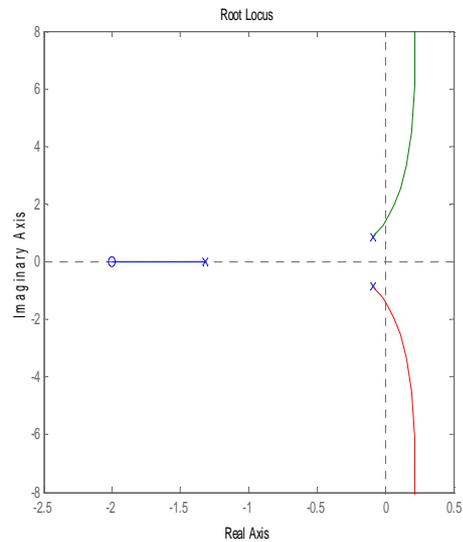


Figure 2. Root locus plot for example 1.

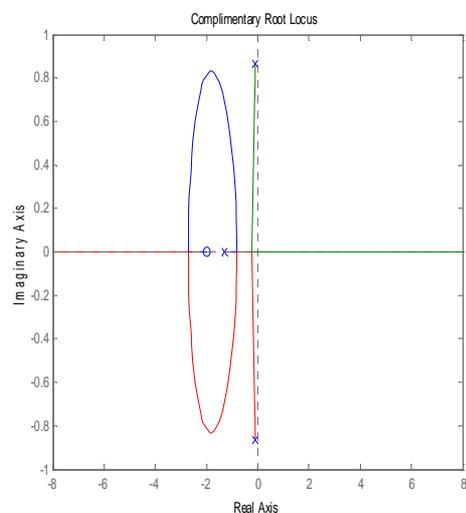


Figure 3. Complimentary Root locus plot for example 1.

**SIMULATED CHARACTERISTICS AND RESULTS**

From our analysis, we have drawn some important results, with the help of MATLAB Version 7.6.0.324 software. From Figure 2,

we got Pole at  $s=0 \pm i1.42$  and overshoot=100% for Gain=2.03 in Root Locus. And from Figure 3, we got Pole at  $s=0$  and overshoot=0% for Gain=1 in Complimentary Root Locus.

	S	K	Overshoot
RL	$0 \pm i1.42$	2.03	100%
CRL	0	1	0%

**Table 1. Gain and Overshoot for RL and CRL.**

It means for moving Poles the system becomes undamped i.e. highly unstable in Root locus and critically damped i.e. highly stable in Complimentary Root Locus. An Important result is that Complimentary Root Locus is more stable than Root Locus.

**CONCLUSION:**

In this paper, we have proposed a method to draw complementary root locus. An interesting by-product of this work is that we found that Overshoot in case of Root Locus is increasing continuously as compared to Complementary Root locus, which clearly shows that CRL is more stable than RL. In the future, we will begin to apply the proposed method in Digital control System and in stability analysis of Oscillators. The properties of complementary root locus in z-plane are identical to that in s- plane except that interpretation must be made with respect to unit circle  $|z|=1$ .

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