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STUDY OF METHOD TO DIFFERENTIATE DISCONTINOUS SIGNAL

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Abstract: In this paper, a method is proposed to find out the first derivative of a discontinuous signal. It is defined that a signal may be discontinuous at more than one point. The basic definition of differentiation is used to find the derivative. The shifting property of signal is used to demonstrate the method. The method has been presented using suitable example. With the help of proposed method, the problems of signals related to discontinuous functions can be easily solved.

Keywords: Discontinuity, Impulse Function, Shifting Property, Derivative.

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INTRODUCTION

The Discontinuous Signal with single or multiple discontinuities was introduced in Calculus. There are many important signals which are discontinuous in nature like unit step function and signum function[1]. The differentiation of discontinuous signal is not an easy task. The transformation techniques are normally used to find out the derivative of discontinuous signals[2].

The basic method of differentiation can also be used to find out the differentiation of discontinuous signal. This method includes simple steps with the help of shifting property of signals[1-2].

Almost all the introductory text books for disciplines about signal system in undergraduate engineering courses present the methods to differentiate continuous signals but don't present information about differentiating discontinuous signal[1-3].

MATERIALS AND METHODS

1) Dirac-Delta Function

Dirac-Delta Function is also called Impulse Function$\delta(t)$. The impulse function can be defined as a signal having infinitesimal width $(\Delta t \to 0)$ and infinite height$(\mathrm{h} \to \infty)$.

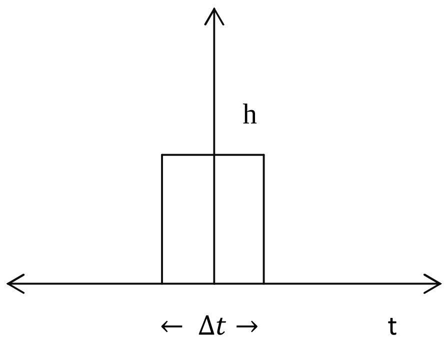

Figure 1. Dirac-Delta Function

The impulse signal is having finite area called the Strength of the signal.

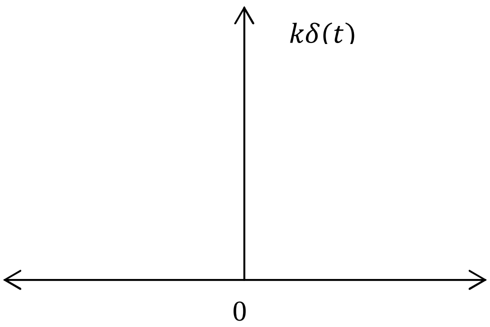

Figure 2. Impulse Function

$$\delta(t) = \begin{cases} k; & t = 0 \\ 0; & \mathrm{t} \neq 0 \end{cases} \qquad (1)$$

$$\text{Area} = \int_{-\infty}^{\infty} \delta(t)dt = \mathrm{k} \qquad (2)$$

Where k represents the strength of the impulse signal.

Unit Impulse Function

If k=1, then the above equation (1) and (2) can be written as:

$$\delta(t) = \begin{cases} 1; & t = 0 \\ 0; & \mathrm{t} \neq 0 \end{cases} \qquad (3)$$

Area = $\int_{-\infty}^{\infty} \delta(t)dt$ = 1 (4)

The resulting signal is called as Unit Impulse signal.

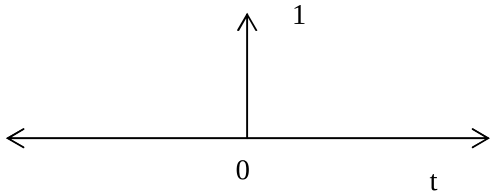

Figure 3.Unit Impulse Signal

Time Shifting Property

The time shifting property of a signal is of two types:

1) Time Advance shifting x(t+a)

2) Time Delay shifting x(t-a)

Where 'a' is a positive integer.

The time shifting property neither affects the width nor the amplitude of the signal. Time Advance means shifting in left side and Time Delay means shifting in right side.

Let x(t)=$\delta(t)$. Then x(t+a)= $\delta(t+a)$ as shown in Figure 4.

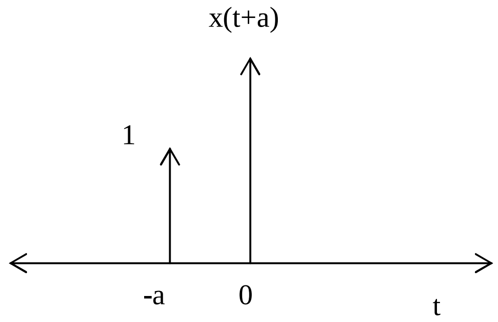

Figure 4. Time Advance Signal

And x(t-a)= $\delta(t-a)$ as shown in Figure 5.

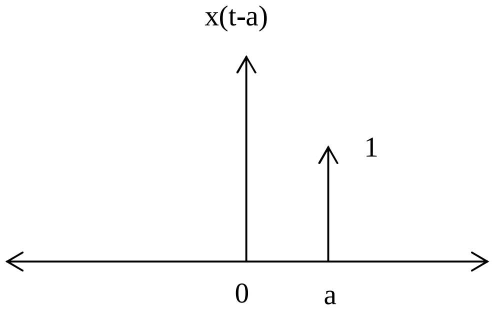

Figure 5. Time Delay Signal

Basic Method of Differentiation:

The Basic method of Differentiation can be defined as

$$y(t) = \frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t)-x(t-\Delta t)}{t-(t-\Delta t)} \quad (5)$$

On simplifying the above equation (5),

$$y(t) = \frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t)-x(t-\Delta t)}{\Delta t} \quad (6)$$

Derivative of Discontinuous Function

(a) let $x_1(t)$ be a discontinuous function with point of discontinuity at t=0, as shown in Figure 6.

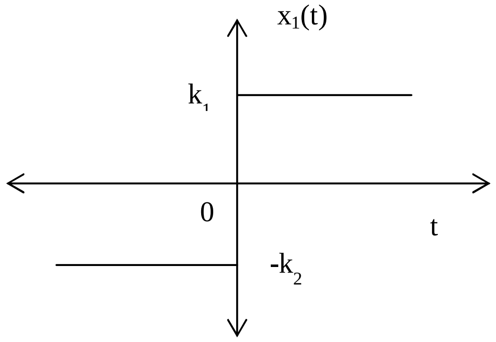

Figure 6. Discontinuous Signal

Where $|k_1|$ may or may not be equal to $|k_2|$

From equations (5) and (6),

$$y_1(t) = \frac{d\,x_1(t)}{dt}$$

$$y_1(t) = \lim_{\Delta t \to 0} \frac{x_1(t) - x_1(t - \Delta t)}{\Delta t} \quad (7)$$

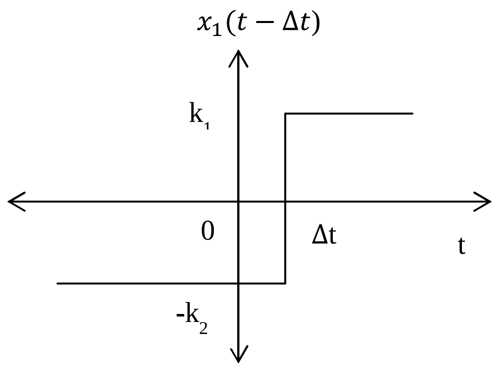

Figure 7. Shifted Discontinuous Signal

By shifting the signal $x_1(t)$ by infinitesimal time Δt the above Figure 7 is obtained.

From equation (7), signal $y_1(t)$ is obtained as shown in Figure 8.

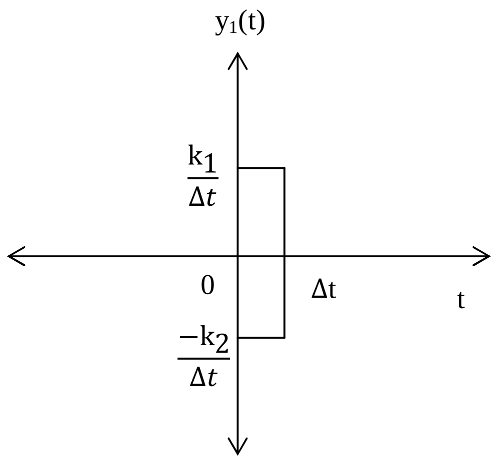

Figure 8. Differentiated Signal

Width of signal $y_1(t)$ is infinitesimal that is $\Delta t \to 0$.

Positive height of the signal is

$|h_+| = \frac{k_1}{\Delta t}$

Negative height of the signal is

$|h_-| = \frac{k_2}{\Delta t}$

Therefore the total height is

$|h| = |h_+| + |h_-|$

$= \frac{k_1}{\Delta t} + \frac{k_2}{\Delta t} = \frac{k_1+k_2}{\Delta t}$

As $\Delta t \to 0$, $|h| \to \infty$.

Area of the signal $y_1(t)=|h| X \Delta t$

Area$=\frac{k_1+k_2}{\Delta t} X \Delta t$

Area$=k_1 + k_2$ (strength of the signal)

From the above analysis it is clear that the resulting signal $y_1(t)$ is an impulse signal with strength equal to total excursion that is from $-k_2$ to k_1.

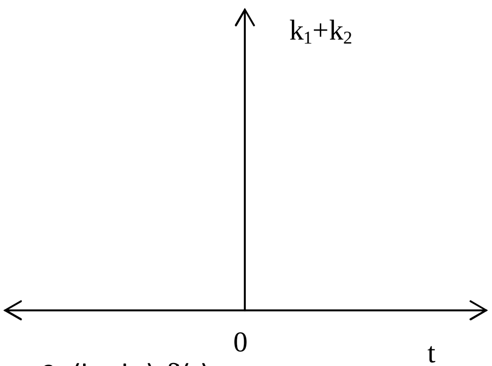

Figure 9. $(k_1+k_2)\ \delta(t)$

(b)) let $x_2(t) = x_1(t-a)$ be a discontinuous function with point of discontinuity at t=a as shown in Figure 10.

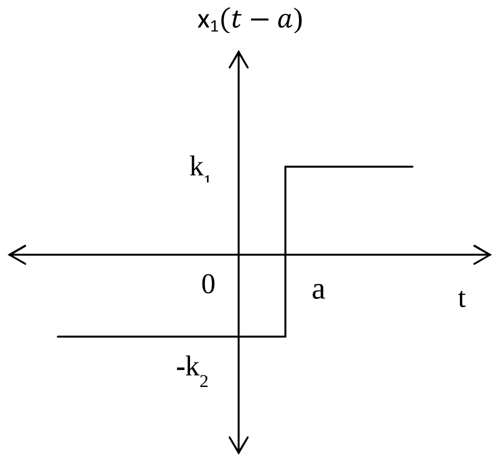

Figure 10. $x_1(t\text{-}a)$ Signal

Where $|k_1|$ may or may not be equal to $|k_2|$

From equations (5) and (6),

$$y_2(t) = \frac{d\, x_2(t)}{dt}$$

$$y_2(t) = \lim_{\Delta t \to 0} \frac{x_2(t) - x_2(t-\Delta t)}{\Delta t} \quad (8)$$

By shifting the signal $x_2(t)$ by infinitesimal time Δt the following Figure 11 is obtained.

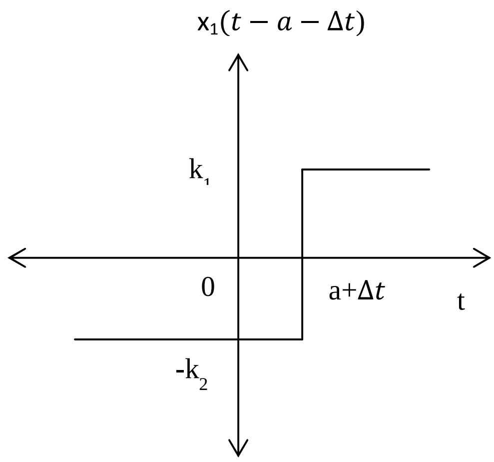

Figure 11. $x_1(t-a-\Delta t)$ Signal

From equation (8) Figure 12 is obtained.

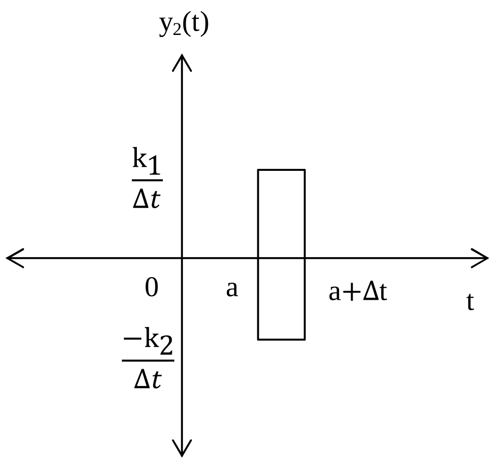

Figure 12. Differentiated Signal

As $\Delta t \rightarrow 0$, |h|→ ∞.

Area of the signal $y_2(t)=|h| X \Delta t$

Area=$\frac{k_1+k_2}{\Delta t}\ X\ \Delta t$

Area=$k_1 + k_2$ (strength of the signal)

From the above analysis it is clear that the resulting signal $y_2(t)$ is an impulse signal with strength equal to total excursion that is from $-k_2$ to k_1 as shown in Figure 13.

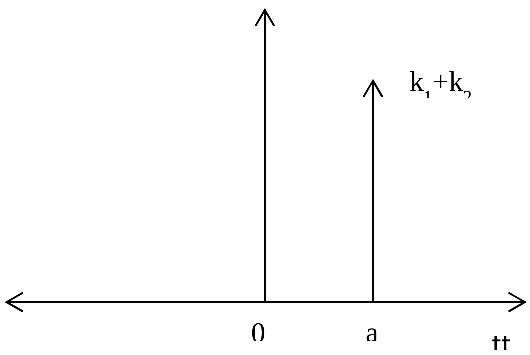

Figure 13. $(k_1+ k_2)\ \delta(t-a)$ Signal

(c) let $x_3(t) = x_1(t+a)$ be a discontinuous function with point of discontinuity at t=-a.

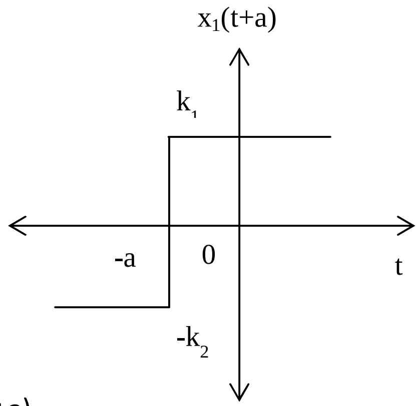

Figure 14. $x_1(t+a)$

From equations (5) and (6),

$$y_3(t) = \frac{d\,x_3(t)}{dt}$$

$$y_3(t) = \lim_{\Delta t \to 0} \frac{x_3(t) - x_3(t-\Delta t)}{\Delta t} \qquad (9)$$

By shifting the signal $x_3(t)$ by infinitesimal time Δt the following Figure 15 is obtained.

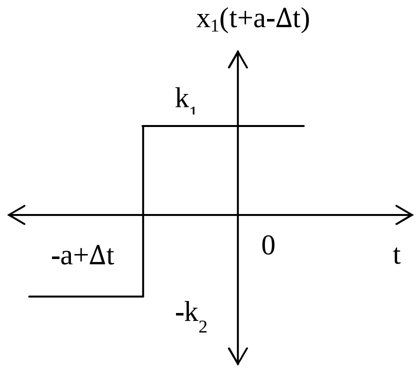

Figure 15. $x_1(t+a-\Delta t)$ Signal

From equation (9) Figure 16 is obtained.

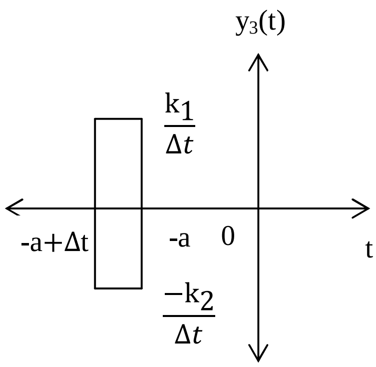

Figure 16. Differentiated Signal

As $\Delta t \rightarrow 0$, $|h| \rightarrow \infty$.

Area of the signal $y_3(t)=|h| X \Delta t$

Area=$\frac{k_1+k_2}{\Delta t} X \Delta t$

Area=$k_1 + k_2$ (strength of the signal)

From the above analysis it is clear that the resulting signal $y_3(t)$ is an impulse signal with strength equal to total excursion that is from $-k_2$ to k_1.

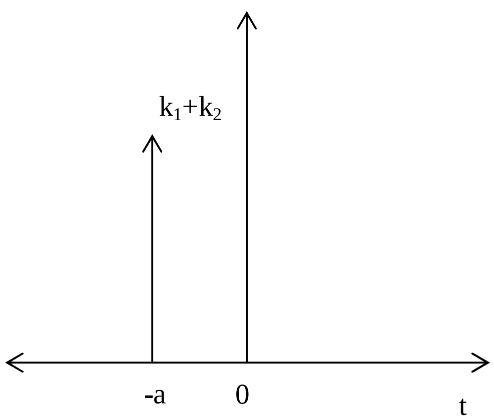

Figure 17. $(k_1+k_2)\ \delta(t+a)$

Example 1

Let k_1=1 and k_2=0; then $x_1(t)=u(t)$

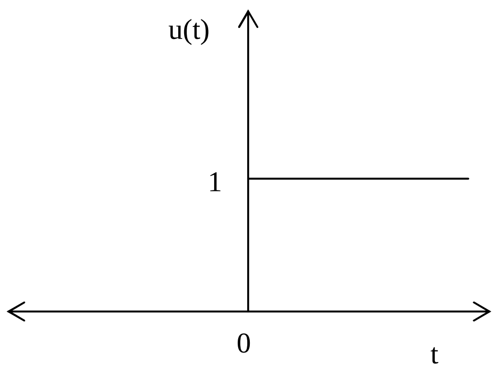

Figure 18. Unit Step Signal

Where u(t) is unit step function which is defined as $u(t) = \begin{cases} 0; & t < 0 \\ 1; & t > 0 \end{cases}$

Shifted Unit Step Signal by infinitesimal width $\Delta t \rightarrow 0$ is as follows:

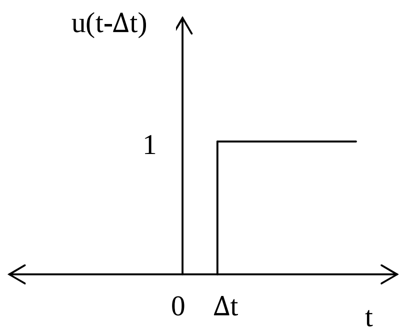

Figure 19. Shifted Unit Step Signal

On applying our method of basic Differentiation, an Impulse signal is obtained at Point of Discontinuity(t=0) with strength equal to unity.

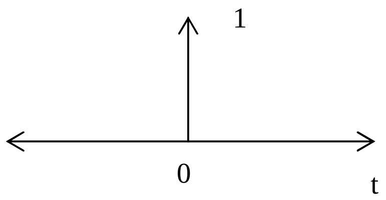

Figure 20. Impulse Signal

Example 2

Let k_1=1 and k_2=1; then x_1(t)=sgn(t)

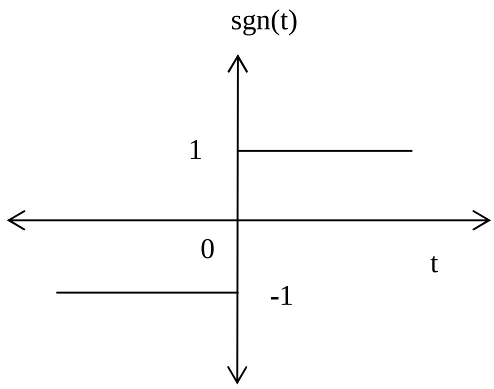

Figure 21. Signum Function

Where sgn(t) is signum function which is defined as $sgn(t) = \begin{cases} -1; & t < 0 \\ 0; & \text{t} = 0 \\ 1; & t > 0 \end{cases}$

Shifted Signum Function by infinitesimal width Δt →0 is as follows:

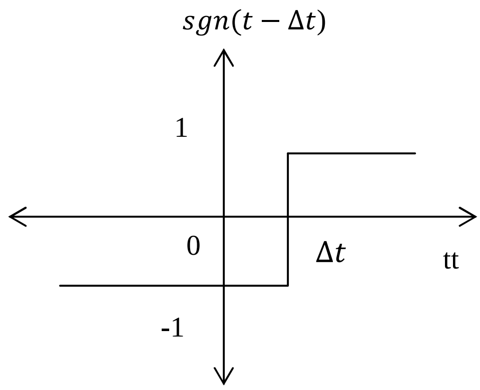

Figure 22. Shifted Signum Function

On applying our method of basic Differentiation, an Impulse signal is obtained at Point of Discontinuity (t=0) with strength equal to two units.

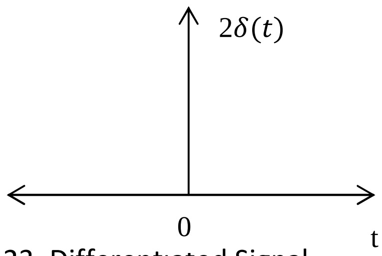

Figure 23. Differentiated Signal

Example 3

Let x(t) be any arbitrary signal with k_1=5 and k_2=3 as shown below

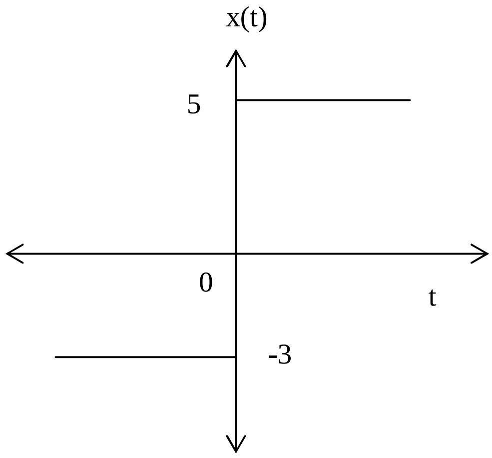

Figure 24. Signal x(t)

By shifting the Signal x(t) by infinitesimal width $\Delta t \to 0$ following Figure is obtained:

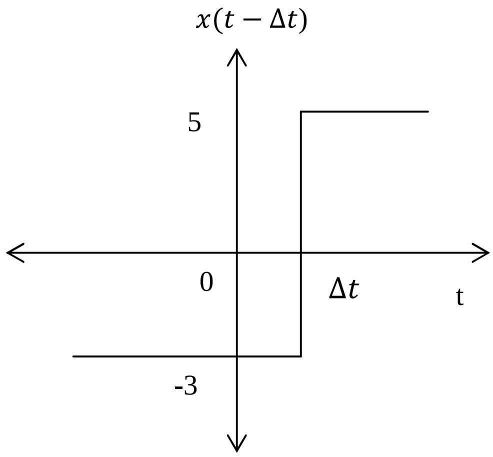

Figure 25. Signal x(t-Δt)

On applying our method of basic Differentiation, an Impulse signal is obtained at Point of Discontinuity (t=0) with strength equal to eight units.

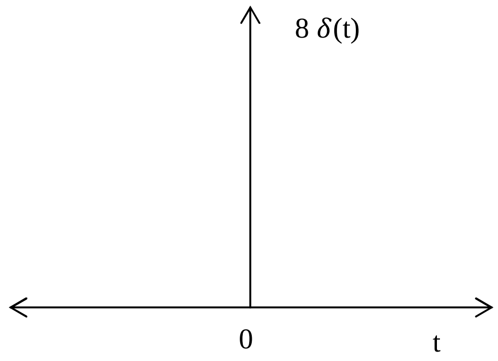

Figure 26. Differentiated Signal

Example 4: To find Probability Distribution Function (PDF) from given Cumulative Distribution Function (CDF) [3].

Probability Distribution Function (PDF) $f_X(x)$ can be defined as

$$f_X(x) = \frac{d\,F_X(x)}{dx}$$

Where $F_X(x)$ is Cumulative Distribution Function (CDF).

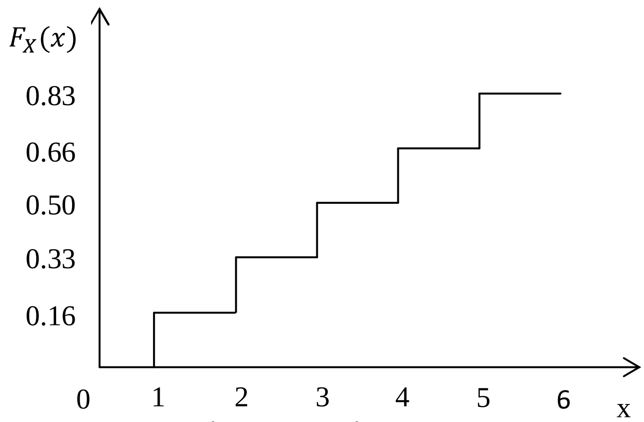

Figure 27. Cumulative Distribution Function

Let $F_X(x)$= P{X$\leq x$}; -∞ $\leq x \leq \infty$

$F_X(x)$ is a discontinuous function with multiple discontinuity points (x=1,2,3,4,5,6) as shown in Figure 27.

By shifting CDF by infinitesimal width $\Delta t \rightarrow 0$ following Figure is obtained:

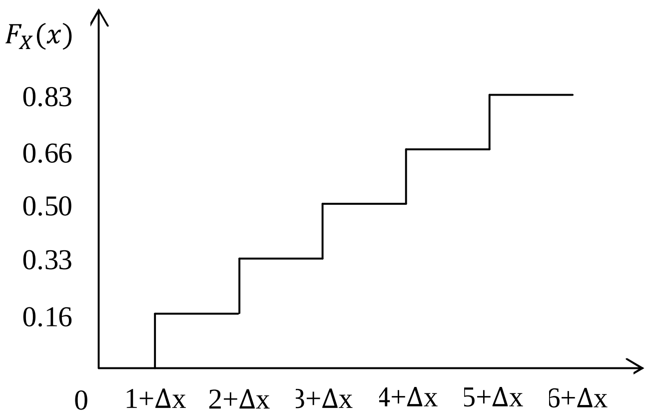

Figure 28. Shifted CDF

On differentiating $F_X(x)$ with the help of proposed method to find PDF, Impulses are obtained at all points of discontinuity with strength equal to total excursion at points of discontinuities as shown below:

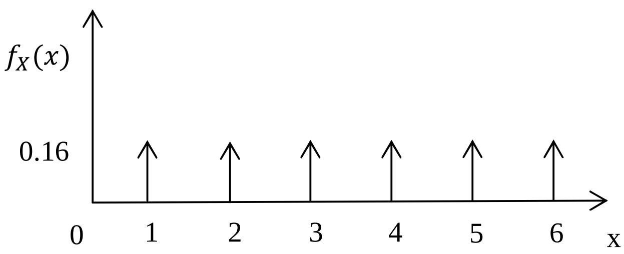

Figure 29. PDF

RESULT

From our analysis we have drawn an important result. We found that on differentiating a discontinuous signal an impulse signal is obtained at the point of discontinuity and the strength of the impulse signal is equal to the total excursion of discontinuous signal at the point of discontinuity. In case of signal having multiple discontinuity, impulses are obtained at each point of discontinuity with strength equal to total excursion at that point of discontinuity.

CONCLUSION

In this paper, we have proposed a method to differentiate the discontinuous signal with the help of basic differentiation method and shifting property of signal.An interesting by-product of this work is that we found thatsame method can be applied to differentiate the discontinuous signal with multiple discontinuities, which find its application in deriving probability distribution function (PDF) from cumulative distribution function (CDF) in communication syste

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