Research Article **Impact Factor: 2.275** **ISSN: 2319-507X**
AK Mishra, IJPRET, 2014; Volume 2 (10): 80-90 **IJPRET**

INTERNATIONAL JOURNAL OF PURE AND APPLIED RESEARCH IN ENGINEERING AND TECHNOLOGY

A PATH FOR HORIZING YOUR INNOVATIVE WORK

A THEORETICAL EVALUATION OF THE BOGOLYUBOV QUASI PARTICLE SPECTRUM IN BOSE LIQUID

A. K. MISHRA[1], S. N. PRASAD[2], L. K. MISHRA[3]

1. Department of Physics, Vidyadan institute of Technology & management, Areon, Buxar (Bihar).
2. Department of Physics, B. D. College, Patna (Bihar).
3. Department of Physics, Magadh University, Bodh-Gaya-824234(Bihar).

Accepted Date: 13/04/2014 ; Published Date: 01/06/2014

Abstract: In this paper, we have theoretically evaluated the Bogolyubov quasi particle spectrum in Bose liquid. Our results were compared with inelastic neutron scattering experiments. Our theoretically evaluated results are in close appearance with hard sphere potential. It was also observed that soft sphere model repulsive potential gives very good agreement between the theoretical quasi particle spectrum and the experimental spectrum of elementary excitations in quantum Bose liquid.

Keywords: Semi empirical potential, quasi particle spectrum, Bose liquid, inelastic neutron scattering experiments, super fluid state, elementary excitation, roton minimum, Shadow wave function

Corresponding Author: MR. A. K. MISHRA

Access Online On:

www.ijpret.com

How to Cite This Article:

PAPER-QR CODE

A. K. Mishra, S. N. Prasad, L. K. Mishra; IJPRET, 2014; Volume 2 (10): 80-90

INTRODUCTION

Although there are large numbers of calculations[1-6] for the spectrum of elementary excitation in this super fluid (SF) ^{4}He Bose liquid but there remains some problem in this direction. There is an excellent agreement with experimental data in the region of the roton minimum obtained by Monte Carlo method making use of the shadow wave function[1] and by the correlation basis function[2]. These calculation also employed the modern inter atomic potentials[3-5] for ^{4}He. There are some calculation using microscopic perturbation theory [6-8] for long wave phonon part of the spectrum $E(p) = C_1 p$ where C_1 is the speed of the first (hydrodynamic) sound in liquid ^{4}He. These calculation faces some principal difficulties become the non-renormalized perturbation theory gives rise to infrared divergence and are non-analytical[9-12] at $p \rightarrow 0$ and ⍰ → 0. These difficulties have been removed by the application of combined variable techniques[13]. These variables reduce to hydrodynamics variable of macroscopic quantum hydrodynamics[14] in the long-wavelength limit ($p \rightarrow 0$). On the other hand in the short wavelength domain they correspond to the bosonic quasi particle creation and annihilation operators.

Neutron inelastic data[15,16] and results in quantum evaporation of ^{4}He atoms[17] show that the maximal density ρ_0 of the single particle BEC (Bose-Einstein Condensate) at $T < T_\lambda$ does not exceed of the total density ρ of the liquid ^{4}He, whereas the density of the SF component $\rho_s \rightarrow \rho$ at $T \rightarrow 0$. This indicates that there is strong interaction between ^{4}He atoms and the quantum structure of the super fluid condensate in He-II carry an excess density $(\rho_s - \rho_0) >> \rho_0$. This requires much more investigations.

In this paper, we have studied various semi empirical potentials to study helium-helium interaction. These potentials involve strong repulsion at small distances and weak Vander Waals attraction at large distances. We have also computed the Bogolyubov spectrum of a dilute quasi ideal Bose gas and compared our evaluated results with inelastic neutron scattering data. Our evaluated results are in good match with the experimental data.

MATERIALS AND METHODS

In order to describe the interaction of He atoms in real space one uses various semi empirical potentials. These potentials involve strong repulsion at small distances and weak Vander Waals attraction at large distances. However, most of those potentials are characterized by strong divergence at $r \rightarrow 0$. For Lennard-Jones Potential[18].

$$U_{LR}(r) = \in\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] \qquad r<r_c \qquad (1)$$

e and σ are potential parameters, if one takes the Fourier transform of the potentials

$$V(p) = \int d^3 r U(r) e^{ip.r}$$
$$= \frac{4\pi}{p}\int_0^\infty rU(r)Sin(pr)dr \qquad (2)$$

One cannot use this potential for the description of pair interaction in momentum space as it is infinite, diverging as the lower limit.

The other potential which is used for the calculation of inter atomic interaction and of possible bond state is Aziz potential[19].

$$U_A(r) = A\exp(-\alpha r - \beta r^2) - \exp[-((r_0/r)-1)^2]\sum_{K=0}^{2} C_{2k+6} r^{-2k-6} \qquad r<r_0$$

$$= A\exp(-\alpha r - \beta r^2) - \sum_{K=0}^{2} C_{2k+6} r^{-2k-6} \qquad r\geq r_0$$

Where,

A = 1.8443101 x 10^5K

α = 10.43329537 $Å^{-1}$

⍰ = 2.27965105 $Å^{-2}$

C_6 = 1.36745214 K x $Å^{-6}$

C_8 = 0.42123807 K x $Å^{-8}$

C_{10} = 0.17473318 K x $Å^{-10}$

Such potentials remain finite at r = 0 due to the non analytic exponential dependence on r which suppresses any power divergence at r → 0. The potential (3) is convenient for calculations in real space making use of Jastrow-like wave functions. But employing its Fourier component for solving nonlinear integral equations is technically difficult. The model potentials which describes interaction of helium atoms in real space takes into account that the distance less than the quantum radius of the helium electron shell r_0 = 1.22 Å the Coulomb repulsion between the nuclei sets in.

Piashitskij etal[20] suggested a potential which diverges as r^{-1} at $r \rightarrow 0$

$$\begin{aligned} U(r) &= \frac{4e^2}{r}(1+\mu r)\exp\left(-\frac{r}{\alpha}\right) && r \le r_c \\ &= \epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] && r > r_c \qquad (4) \end{aligned}$$

From the conditions of continuity of the potential U(r) and its first derivatives at $r = r_c$, one determine the value of the parameters r_c= 2.38Å and $\mu = 229$ $Å^{-1}$ for $\alpha = r_0/2 = 0.61$ Å ε = 10.8 K and σ = 2.642 Å.

In many body problem, the quantum effects play an essential role and in that case the two-body potential is irrelevant. For that one should have a model potential with a simpler analytic expression. There is a model potential in the form of a Fermi type function in real space

$$U_F(r) = U_0\left[\exp\left(\frac{r^2 - a^2}{b^2}\right) + 1\right]^{-1} \qquad (5)$$

Which at b = 0 degenerates into a state function at θ (a-r) with a finite height U_0 at $r < a$. This model corresponds to a model of 'soft' sphere. Its Fourier component is expressed in terms of first order spherical Bessel function[21]

$$V(p) = V_0\frac{j_1(pa)}{pa},\ j_1(x) = \frac{Sin(x) - xCos(x)}{x^2} \qquad (6)$$

Where $V_0 = 3V(0) = 4$⍰ $U_0\, a^3$. It is an oscillating sign changing function of momentum transfer p. The same oscillating Fourier component is characteristics of smooth potential V(r) in the form of Leonhard type function[22] having an infinite negative derivatives at the inflection point r = a.

$$U_L(r) = \frac{U_0}{2}\left[1 + \frac{1 - \frac{r^2}{a^2}}{\frac{2r}{a}} ln\left|\frac{a+r}{a-r}\right|\right] \qquad (7)$$

Formally this problem is an inverse to one of periodic oscillations of spin density in real space of the exchange interacting spins of electrons in metal. This is popularly known as RKKY (Ruderman-Kittel-Kaisuya-Yosida) oscillation[23]. The same behavior is characteristics of the Fourier components of more realistic potentials diverging faster than r^{-1} at $r = 0$. It posses inflection points in the radial dependence at $r = r_c$. The amplitude of the oscillation of the Fourier component of the Fermi type potential (5) at $b \neq 0$ is falling off exponentially with the increase of the parameter b due to the decreasing absolute value of the negative derivative at the inflection point.

If one substitutes the oscillating potential (6) into the Bogolyubov of a dilute quasi-ideal Bose-gas[24,25].

$$E_B(p) = \left\{\frac{p^2}{2m}\left[\frac{p^2}{2m} + 2nV(p)\right]\right\}^{1/2} \qquad (8)$$

RESULTS AND DISCUSSION

In this paper in order to describe the interaction of He atoms in real space, various empirical potentials are used which involve strong repulsion at small distances and week Vander Waals attraction at large distances. Most of the potentials are characterized by a strong divergence at $r \rightarrow 0$. Among the various potential discussed, there is a repulsive potential in the frame work of 'soft spheres model' the Fourier component v(p) is an oscillating sign - changing function of momentum transfer p. It appears that in a certain region of momentum space at $p \neq 0$ there is an effective attraction between bosons $v(p) < 0$. This has nothing to do with Vander Waals forces and has a quantum mechanical diffraction nature. This attraction gets substantially enhanced due to the multi particle effects of renormalization (screening) of the initial interaction[28-37].

We have theoretically compared the Bogolyubov quasi particle spectrum equation (8), with the oscillating Fourier component [$v(p) = V_0 \, Sin\,(pa) / pa$] and also with experimental spectrum[26,27].

The results are in close appearance with the hard sphere potential (equation 6). The results are shown in table T_5. In table T_1 we have shown the evaluated radial dependent of potential (4) and Aziz potential (5), in table T_2 the two forms of finite potential (5) has been shown in real space (r/a) keeping b = 0 [I - form] and b = 0.5a [II - form]. In Table T_3, we have given the (Lindhardt function potential [$U_L(r)$ / Uo] shown in equation (7) in real space (r/a). In Table T_4, we have shown the momentum dependent of the Fourier component of the Fermi type potential (5) for b = 0 and potential (4).

From these studies it looks that the soft sphere model repulsive potential which gives very good agreement between the theoretical quasi particle spectrum $E_B(p)$ and the experimental spectrum of elementary excitation in quantum Bose liquid ^{4}He, is much smaller than the value of the Aziz-type potential at $r \rightarrow 0$. It is a result of strong quantum diffraction effects in Bose liquids, because the average distance between particles is equal or less than the de Broglie wavelength for bosons.

CONCLUSION:

In this paper, we have theoretically evaluated Bogolyubov quasi-particle spectrum in Bose liquid and the results were compared with inelastic neutron scattering experiments. Our theoretically evaluated results are in close appearance with hard sphere potential. It was also seen that soft sphere model repulsive potential obtained results for quasi-particle spectrum are smaller than the value of the Aziz-type potential at r=o. This shows strong quantum diffraction effects in Bose liquids.

Table T_1

An evaluated result of radial dependence of potential (4) and Aziz potential (3)

r (Ǻ)	Potential (4) U (r) (K)	Aziz Potential U_A (r) (K)
2.5	105.2	110.8
2.6	95.6	106.5
2.7	84.3	98.2
2.8	50.6	68.6
2.9	16.7	23.2
3.0	-12.2	-16.8
3.1	-14.6	-15.6
3.2	-10.4	-11.8

3.3	-8.6	-9.9
3.4	-6.5	-7.3
3.5	-5.6	-6.8
4.0	-4.32	-5.8
4.5	-4.00	-3.8
5.0	**-0.08**	**-1.02**

Table T_2

An evaluated result of the finite potential model (5) in real space (r/a) [I – form for b=0] and [II – form for b=0.5a.]

r/a	$U_F(r)$ / U_0 for b = 0	$U_F(r)$ / U_0 for b=0.5a
0.0	1.00	1.00
0.1	1.00	0.998
0.2	1.00	0.996
0.3	1.00	0.987
0.4	1.00	0.965
0.5	1.00	0.928
0.6	1.00	0.889
0.7	1.00	0.793
0.8	1.00	0.786
0.9	1.00	0.684
1.0	0.00	0.546
1.1	0.00	0.253
1.5	0.00	0.446
1.6	**0.00**	**0.054**

Table T_3

An evaluated results of the Lind hart function potential $U_L(r)$ / U_0 equation (7) in real space r/a.

r/a	$U_L(r)$ / U_0
0.0	1.00
0.1	0.984
0.2	0.975

0.3	0.952
0.4	0.886
0.5	0.825
0.6	0.795
0.7	0.705
0.8	0.659
0.9	0.613
1.0	0.526
1.1	0.508
1.2	0.432
1.3	0.326
1.4	0.286
1.5	0.239
1.6	0.187
1.8	**0.144**

Table T_4

An evaluated results of the momentum dependence of the Fourier components of the Fermi type potential (5) for b = 0 and potential (4).

	-----------------V(p) / V(o) -----------------	
p ($Å^{-1}$)	Potential (5)	Potential (4)
0.0	1.0	1.0
0.2	0.869	0.986
0.4	0.792	0.924
0.5	0.607	0.885
0.6	0.496	0.816
0.8	0.288	0.774
1.0	0.087	0.625
1.5	-0.156	0.463
2.0	-0.108	0.136
2.5	-0.088	-0.115
3.0	0.145	-0.056
3.5	0.208	0.069

4.0	0.125	0.145
4.5	0.086	0.186
5.0	**-0.095**	**0.068**

Table T_5

An evaluated result of Bogolyobov quasi particle spectrum (equation 8) with the oscillating Fourier component of the hard sphere potential equation (6)

p (Ǻ$^{-1}$)	**------------------E_B(p), K ------------------**	
	Hard sphere Potential eq^n (6)	Bogolyobov quasi particle Spectrum eq^n (8)
0.0	0.052	0.046
0.25	3.628	4.554
0.50	6.836	7.397
0.75	10.957	11.412
1.00	14.254	15.350
1.25	12.868	13.145
1.50	10.296	11.254
1.75	8.144	9.266
2.00	9.686	10.052
2.25	11.234	12.645
2.50	**16.286**	**17.128**

REFERENCES:

1. J. Boronot and J. Casulleras, Europhys. Lett. 38, 291 (1997)

2. E. Krotscheck, M. D. Miller and R. Zillich, Physica B 280, 59 (2000)

3. R. A. Aziz and M. J. Slaman, J. Chem. Phys. 94, 8047 (1991)

4. A. R. Janzen and R. A. Aziz, J. Chem. Phys. 103, 9626 (1995)

5. T. Korona, H.L. Williams, R. Bukorski, B. Jezorski and K.Szalewicz, J. Chem. Phys. 106, 5109 (1997)

6. S. T. Belyaev, JETP 34, 417-433 (1958)

7. A. A. Abrikosov, L. P. Gorkov and I. E. Dzyalopshinskij, Methods of Quantum Filed Theory in Statistical Physics, Prentic Hall, Englawood cliffs, N. J. Dover, New York (1963)

8. P. Nozieres and D. Pines, Theory of Quantum Liquids, Academic, New York (1969)

9. A. A. Nepomnyaschii and Y. A. Nepomnyaschii, JETP, Letters 21, 3 (1976)

10. Y. A. Nepomnyaschii and A. A. Nepomnyaschii, JETP, 48, 493 (1978)

11. V. H. Popov and V. A. Serednyakov, JETP, 50, 193 (1979)

12. Y. A. Nepomnyaschii, JETP, 89, 511 (1985)

13. V. H. Popov, Continuum Integrals in Quantum Field Theory and Statistical Physics (Nauka), Moscow (1973)

14. L. D. Landau, JETP, 17, 91 (1947)

15. T. R. Sosnic, W. M. Snow and P. E. Sokol, Phys. Rev. 41, 11185 (1990)

16. B. Fak and J. Bossy, J. Low. Temp. Phys. (JLTP) 113, 531 (1998)

17. A. F. G. Wyatt, Nature 391, 56 (1998)

18. T. R. Koehler, Phys. Rev. Lett. (PRL) 17, 89 (1966)

19. S. V. Mashkevich and S. I. Vilchynsky, J. Mol. Liquids 93, 81 (2001)

20. E. A. Pashitskij, S. V. Mashkevich and S. I. Vichynsky, Phys. Rev. Lett. (PRL) 89, 075301 (2002)

21. E. A. Pashitskij, Ukr. Fiz. Journal 18, 1439 (1973)

22. R. White, Quantum theory of magnetism, MIR, Moscow (1985)

23. E. A. Pashitskij and S. V. Mashkevich , Low. Temp. Phys. 27, 185 (2001)

24. N. N. Bogolyubov, IZV, AN, USSR, Seriyh Physica 9, 23 (1947)

25. K. A. Bruckner & K. Sawada Phys. Rev. 106, 1117, 1128 (1957)

26. S. Putterman, P. H. Roberts, C. A. Joans & A. Larraza, In excitations in two dimensional and three dimensional quantum Fluids, A. F. G. Wyatt and H. J. Lanteo (Eds) Plenum, New York (1991)

27. K. H. Anderson, W. G. Stirling, R. Scherm, A. Stanault, B. Fak, H. Godfrin & A. J. Dianoux, J. Phys. Condensed Matter 6, 821 (1994)

28. E. A. Pashitskij, S. V. Mashkevich and S. I. Vichynsky, Low. Temp. Phys. 28, 47 (2002)

29. E. A. Pashitskij, S. V. Mashkevich and S. I. Vichynsky, J. Low. Temp. Phys. (JLTP) 134, 851 (2004)

30. A. Cheumachenko & S. I. Vichynsky, J. Mol. Liquids, 124, 72 (2006)

31. M. Iskin and C. A. R. Sade Melo, Phys. Rev A76, 013601 (2007)

32. T. Ohashi, T. Momoi, H.Tsunethgu and N. Kawakami, Phys. Rev. Lett. (PRL) 100, 076402 (2008)

33. R. M. Lutechyn, P. Nagorukl and V. M. Yakovenko, Phys. Rev. B80, 104508 (2009)

34. W. Ketterle, Y. Shin, A. Schivoten and C. H. Schunk., Int. J. Mod. Phys. B24, 3950 (2010)

35. L. Rodzihovsky, arXiv: 1102. 4903V1 [Cond-mat. Quan. Gas] 24 Feb 2011

36. D. Blanc, K. M. daily and D. Rakshit, Phys. Rev. A85, 021601 (2012)

37. B. S. Rem, A. T. Grider, U. Eisnam, T. Langen and C.Solomon, Phys. Rev. Lett.(PRL) 110, 163202 (2013)