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A PATH FOR HORIZING YOUR INNOVATIVE WORK

THE EQUITABLE DOMINATION POLYNOMIAL OF GRAPHS

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Abstract: - One of the Algebraic graph representations is the represent the graph by a polynomial. Recently domination polynomial is introduced and studied in [1]. In this article we introduced a new graph polynomial called equitable domination polynomial, the exact equitable domination polynomial for some standard graphs and some graphs which coming by some operation between graphs like bi-star, spider graph, corona product, friendship graph and book graph. Some coefficient properties of equitable domination polynomial are obtained.

Keywords: Equitable domination, Graph polynomial, Domination polynomial, Equitable domination of Graphs

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INTRODUCTION

In this paper, by graph, we mean finite and undirected with no loops and multiple edges. As usual $p = |V|$ and $q = |E|$ denote the number of vertices and edges of a graph G, respectively. We use $\langle X \rangle$ to denote the sub graph induced by the set of vertices X and by $N(v)$ and $N[v]$ to the open and closed neighborhoods of a vertex v, respectively.

One of the fastest developing areas in theory of graphs is the study of domination and related subset problems. There are many type of dominating set depending about how the author defines the adjacency between vertices.

A set $S \subseteq V$ is called a dominating set of G if every vertex in $V \setminus S$ is adjacent to at least one vertex of S, in another meaning a set S in a graph G is a dominating set of G if $N[S] = V(G)$ and the minimum cardinality of a dominating set is called the domination number of G and is denoted by $\gamma(G)$. A minimum dominating set of a graph G is called a γ-set of G.

For more details about dominating set and domination number we refer the reader to [4].

Recently the equitable dominating set and the equitability in graphs has been introduced and studied in [2,6]. For any two vertices u and v we say that u is equitable adjacent to v if they are adjacent and $|\deg(u) - \deg(v)| \leq 1$. The equitable neighborhood of the vertex v denoted by $N_e(v)$ is defined as $N_e(v) = \{u \in V | u \in N(v), |\deg(u) - \deg(v)| \leq 1\}$. A set $S \subseteq V$ is called an equitable dominating set of G if every vertex in $V \setminus S$ is equitable adjacent to at least one vertex of S. in another meaning a set S in a graph G is an equitable dominating set of G if $N_e[S] = V(G)$ and the minimum cardinality of an equitable dominating set is called the equitable domination number or degree equitable domination number of G and is denoted by $\gamma_e(G)$.

The graph can be studied by studied its representation. There are many ways to represent the graph like drawing, matrices, codes and polynomial. One of the algebraic graph theory branches is graph polynomials and there are many graph polynomials that have been introduced and studied widely like Chromatic polynomials count the number of proper colorings of a graph. Matching polynomials count the number of matching set in the graph. Independence polynomials are generating polynomials for the number of independent sets of each cardinality.

In [1] domination polynomial in graphs is introduced to count the number of dominating sets in a graph of different sizes. And the definition of domination polynomial in graph and the equitable domination in graph motivated us to introduce and study the equitable domination polynomial in graphs.

2 Equitable Domination polynomial of Graphs.

Definition 2.1. *Let $G = (V, E)$be agraph an Equitable Domination polynomial of G denoted by $D_e\,(G, X)$ and defined as: $D_e\,(G, X) = \sum_{i=\gamma_e\,(G)}^{n} D_e\,(G, i)\, x^i$, where n is the number of vertrices inG, $\gamma_e\,(G)$ is the Equitable Domination number of G and $d_e\,(G, i)$ is the number of equitable dominating set in G of size i.*

The roots of the polynomial$D_e\,(G, X)$ is called the equitable domination root of G and denoted by $Z(D_e\,(G, X)$.

Example 2.2. *Let G be a graph as in figure 1*

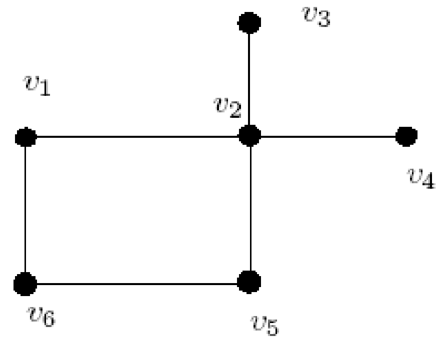

Figure 1

Clearly $\gamma(G) = 2, \gamma_e\,(G) = 4$. First to get to get the domination polynomial $D(G, x)$, we have the dominating of size two. There are three dominating sets of size two which they are $\{v_2, v_6\}, \{v_2, v_5\}, \{v_2, v_1\}$. $d(G, 2) = 3$, the dominating sets of size 3, $\{v_3, v_4, v_6\}$ and every dominating set size two can extend to dominating set of size three in four ways that means $d(G, 3) = 13 - 3 = 10$. because three sets will repeat in another word the dominating sets of size three in G denoted $D(G, i)$ are $\{v_1, v_2, v_5\}, \{v_2, v_5, v_6\}, \{v_2, v_{3,} v_5\}, \{v_2, v_4, v_5\}, \{v_1, v_2, v_6\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}$,

$\{v_2, v_3, v_6\}, \{v_2, v_4, v_6\}, \{v_3, v_4, v_6\}$

Therefore $d(G, 3) = 10$. similarly we can see that there are 19 dominating sets of size 4, $d(G, 3) = 19$ and there are six dominating sets of size five and one dominating set of size six. i.e., $d(G, 5) = 6\ and\ d(G, 6) = 1$. Hence, $D(G, x) = x^6 + 6x^5 + 19x^4 + 10x^3 + 3x^2$ now to get the equitable domination polynomial of G as we saw $\gamma_e\,(G) = 4$ and there isolated vertices

$v_{2,} v_{3,} and\ v_4$ must belong to every equitable dominating set. $D_e(G,4) = \{v_{2,} v_{3,} v_{4,} v_6\}$ there is only one equitable dominating set of size four. There are three equitable dominating sets of size five. $D_e(G,5) = \left\{\{v_2, v_{3,} v_{4,} v_{6,} v_1\}, \{v_{2,} v_3, v_4, v_6, v_5, \{v_{2,} v_{3,} v_4, v_1, v_5\ \}\right\}$ and there is only one equitable dominating set of size six. Hence, $D_e(G,x) = x^6 + 3x^5 + x^4 = x^4(x^2 + 3x + 1)$.Also to get the equitable dominating roots of G we have to solve the equation $x^4(x^2 + 3x + 1) = 0$. That means the roots are $0, -\frac{1}{2}\sqrt{5} - \frac{3}{2}\ and\ \frac{1}{2}\sqrt{5} - \frac{3}{2}$.

Theorem 2.3. *Let $G \cong G_1 \cup G_2$ for any two graphs $G_1 and\ G_2$ Then $D_e(G,x) = D_e(G_1, X)D_e(G_2, X)$.*

Proof: Let G_1 has n_1 vertices and $\gamma(G_1) = \gamma_1\ and\ G_2\ has\ n_2$ vertices and $\gamma(G_2) = \gamma_2$. Clearly $\gamma_e(G_1 \cup G_2) = \gamma_e(G_1) + \gamma_e(G_2)$.Therefore for any dominating set of size $k \geq \gamma_1 + \gamma_2$ is constructed by selecting a dominating set of size J from G_1 where $J \in \{\gamma_1, \gamma_1 + 1, \gamma_1 + 2, \dots \dots \dots ., n_1\}$ and a dominating set of size $k - J$ from G_2, and obviously the number of ways of constructing over all $J = \gamma_1, \gamma_1 + 1, \gamma_1 + 2, \dots \dots \dots ., n_1$ is equal to the coefficient of x^kin $D_e(G_1, X)D_e(G_2, X)$.Hence $D_e(G,x) = D_e(G_1, X)D_e(G_2, X)$.Theorem 2.3 can be generalized in the following result.

Theorem 2.4. *For any graphs $G_{1,} G_{2,} G_{3,} \dots \dots .. G_m\ if\ G = \bigcup_{i=1}^{m} G_i$ then $D_e(G,x) = \prod_{i=1}^{m} D_e(G_i, x)$.* ________________(∗)

Proof: We can prove the theorem by mathematical induction. The result is correct if $m = 2$ by using theorem 2.3., now suppose that $D_e(G,X) = \prod_{i=1}^{k} D_e(G_i, x)$ is true for $m = k$. For some positive integer $2 < k$, that means $D_e(G,x) = \prod_{i=1}^{k} D_e(G_i, x)\ if\ G = \bigcup_{i=1}^{k} G_i$. To prove that equation (∗) is true for $k + 1$. Suppose that $G = \bigcup_{i=1}^{k+1} G_i$. Then $G = \bigcup_{i=1}^{m} G_i \cup G_{k+1}$. By theorem 2.3 $D_e(G,x) = D_e\left(\bigcup_{i=1}^{k} G_i\ , x\right)D_e(G_{k+1}, X)$ and since the equation is true for $m = k$. $D_e(G,X) = D_e(G_{k+1}, x) \prod_{i=1}^{k} D_e(G_i, x) = \prod_{i=1}^{k+1} D_e(G_i, x)$.

Corollary 2.5. *For any totally disconnected graph $\overline{k_n}$, we have $D_e\left(\overline{k_n}, x\right) = x^n$.*

Proof: Clearly $\overline{k_n}$ can be written as nk_1, that means union of n copy of k_1 and by Theorem 2.4,$D_e\left(\overline{k_n}, x\right) = D_e\left(k_1, x\right) \dots \dots \dots \dots .. D_e\left(k_{1,} x\right)$---n times and clearly , for k_1 there is only one equitable dominating set of size one,that means $D_e\left(k_{1,} x\right) = x$. Hence $D_e\left(\overline{k_n}, x\right) = x^n$.

Theorem 2.6. *For any k-regular graph or $(k, k + 1)$ bi-regular graph G, $D(G,x) = D_e(G,x)$.*

Proof: Clearly if G is k-regular graph or $(k, k + 1)$ bi-regular graph then $|\deg(u) - \deg(v)| \leq 1$ for any two vertices u and $v\ in\ G$ .That means any two adjacent vertices in G are also equitable adjacent in G, then $\gamma_e(G) = \gamma(G)$ and any dominating set of G is also equitable dominating set of G. Therefore $D_e(G, i) = d(G, i)$ for any $i = 1,2,3, \ldots \ldots .. n$.Hence$D(G, x) = D_e(G, x)$.

Proposition 2.7. *Let G_1 and G_2 be any two regular grphs of degree k and n_1, n_2 vertices respectively such that $|n_1 - n_2|$. Then $D_e(G_1 + G_2) = \big((1 + x) - 1\big)\big((4x) - 1\big) + D_e(G_1, x) + D_e(G_2, x)$.*

Proof: By the definition of $G_1 + G_2$ any vertex in G_1 will be of degree $k + n_2$ in $G = G_1 + G_2$and any vertex of G_2 will be of degree $k + n_1$ in $G = G_1 + G_2$, therefore any vertex in $G = G_1 + G_2$ will be of degree $k + n_1$ or of degree $k + n_2$ and $|(k + n_1) - (k + n_2)| = |n_1 - n_2| \leq 1$ that means any adjacent vertices in $G = G_1 + G_2$ will be equitable adjacent. Therefore $D_e(G_1 + G_2, x) = D(G_1 + G_2, x)$ and by Theorem [1]. We will get$D_e(G_1 + G_2, x) = ((1 + x)^{n_1})((1 + x)^{n_2} - 1) + D_e(G_1, x) + D_e(G_2, x)$.

Corollary 2.8. *For any complete bi-partite graph $k_{S,t}$*

$$D_e\big(k_{S,t}\,, x\big) = \begin{cases} x^{S+t} & if \quad |S - t| \geq 2; \\ (1 + x)^{S+t} - (1 + x)^S + (1 + x)^t + 1 + x^S + x^t & if \quad |S - t| \leq 1. \end{cases}$$

Proof: (i) if $|S - t| \geq 2$, then all the vertices of $k_{S,t}$ are equitable isolated vertices and in this case $\gamma_e\big(k_{S,t}\big) = S + t$ so there is only one equitable dominating set of size $S + t$. Therefore $D_e\big(k_{S,t}\,, x\big) = x^{S+t}$.

(ii) if $|S - t| \leq 1$,then any two adjacent vertices are also equitable adjacent vertices and it is well known that $k_{S,t} \cong \overline{k_s} + \overline{k_t}$, so by using Proposition 2.7 we get $D_e\big(k_{S,t}\,, x\big) = ((1 + x)^S - 1)((1 + x)^t - 1) + x^s + t$.Hence,$D_e\big(k_{S,t}\,, x\big) = (1 + x)^{S+t} - (1 + x)^S + (1 + x)^t + 1 + x^S + x^t$.

Corollary 2.9. *For any star $k_{1,t}$, $D_e\big(k_{1,S}\,, X\big) = x^t + x(1 + x)^t$.*

Proof: The proof is straightforward by corollary 2.8 by putting$S = 1$.

Proposition 2.10. *Let $G \cong W_n$, wher W_n is wheel of $n \geq 4$ vertices. Then*

$$D_e(G, x) = \begin{cases} x(1 + x)^{n-1} + D(C_{n-1}, x) & if \ n = 4,\ 5; \\ x\big(D(C_{n-1}, x)\big) & if \quad n \geq 6 \end{cases}$$

Proof: It is well known that $W_n = k_1 + C_{n-1}$ and in case W_n has four or five vertices we have the following in case four vertices $W_4 \cong k_4$ and if $n = 5$ then $W_n$is (3,4) semi regular. That means in the two cases $n = 4,5$ any two adjacent vertices are equitable adjacent and then $D_e(G,x) = D(G,X) = x(1+x)^{n-1} + D(C_{n-1},x)$ by Theorem [ ].Second case if $n \geq 6$, then the center of the wheel is equitable isolated vertex and must be belong to every equitable dominating set. Also $\gamma_e(W_n) = 1 + \gamma_e(C_{n-1})$ in this case.Hence for any $i = 1,2,3, \dots \dots \dots .. n$, we have $d_e(G,i) = d_e(C_{n-1},i)$. Therefore$D_e(G,x) = x\big(D(C_{n-1},x)\big) = xD(C_{n-1},x)$.

Proposition 2.11. *let* $G \cong k_1 + H$ *.For any graph* H *with* $\Delta(H) \leq n-3$ *where* $n = |V(H)|$ *and* $\delta(H) \geq 2$*.Then ,* $D_e(G,x) = xD_e(H,x)$.

Proof: Let H be a graph with $\Delta(H) \leq n-3$ *and* $\delta(H) \geq 2$, then , $k_1 + H$ is the joint graph and the vertex in k_1 will have degree n that means $\Delta(G) = n$ if the vertex v is the vertex in k_1 then $\Delta(G) = deg_G(V)$ and clearly since $\Delta(H) \leq n-3$ the vertex v will be equitable isolated vertex in $k_1 + H$ and v must belong to every equitable dominating set of G and $d_e(G,i) = d_e(H,i)$ for any $i \in 1,2,3 \dots \dots .. n$. Therefore $D_e(G,x) = xD_e(H,x)$.

From the definition of equitable domination polynomial, the proof of the following theorem is straightforward'

Theorem 2.12. *Let* $G = (V,b)$ *be a graph with n vertices. Then*

i. *If* G *is equitable connected, then* $d_e(G, n-1) = n$
ii. $d_e(G,x)$ *has no constant term.*
iii. $d_e(G,x)$ *is strictly increasing function in [0,∞).*
iv. *For any equitable induced subgraph of* G *,* $deg(D(G,x)) \geq deg(D(H,x))$.
v. *Zero is equitable domination root of any equitable domination polynomial* $D_e(G,x)$ *with multiplicity* $\gamma_e(G)$.

Theorem 2.13. *For any graph* G*, with n vertices,* $d_e(G, n-1) = n - t$*, where* t *is the number of equitable isolated vertices in* G.

Proof: Let G be a graph with n vertices and t equitable isolated vertices to get the number of equitable dominating sets of size $n-1$,clearly For any vertex v,where $deg_e(v) \geq 1$ *we have* $V - \{v\}$ ia an equitable dominating set of size $n-1$. Therefore $d(G, n-1)$ is equal to the number of vertices which has equitable degree equal or greater than one. So if $t =$

$|\{v \in V(G): deg_e(v) = 0|$ that means the number of equitable isolated vertices in G. Then $d_e(G, n-1) = n - t$.

Proposition 2.14. *For any graph G with n vertices, $D_e(G,x) = x^n$ if and only if G is totally equitable disconnected graph*.

Proof: Let G be a totally equitable disconnected graph. Then for any vertex $v \in V(G), deg_e(v) = 0$ *that means* $\gamma_e(G) = n$. Therefore $D_e(G,x) = x^n$. Conversely, let G be a graph such that $D_e(G,x) = x^n$, where n is the number of vertices in G. Then $d_e(G,n) = 1$, and $d_e(G,i) = 0$ for all $i < n$ that means $\gamma_e(G) = n$. Hence all the vertices in G are equitable isolated vertices. Hence, G is totally equitable disconnected graph.

Proposition2.15. *Let* $G \cong Ho\, k_1$ *be a graph , such that* $\delta(H) \geq 2$ *and* $|V(H)| \geq 3$. *Then* $\gamma_e(G) = |V(H)| + \gamma_e(H)$.

Proof: Let H be a graph with $p \geq 3$ vertices and $\delta(H) \geq 2$. Then the pendant vertices of $G \cong Ho\, k_1$ are equitable isolated vertices and must belong to every dominating set. Hence, any minimum equitable dominating set of H suppose we have $D \cup A$ is minimum equitable dominating set of $\cong Ho\, k_1$, where A is the est of pendant vertices in G.Therefore $\gamma_e(G) = |D| + |A| = \gamma_e(H) + P$. Thus, $\gamma_e(G) = |V(H)| + \gamma_e(H)$.

We can generalize Proposition 2.15 as the following Theorem.

Theorem 2.16. *Let* $G \cong Ho\overline{k_m}$, *be a graph, such that* $\delta(H) \geq 2$ *and* H *has p vertices where* $p \geq 3$. *Then* $\gamma_e(G) = mp + \gamma_e(H)$.

Theorem 2.17. *Let* $G \cong Hok_{1,}$ *where* H *is a graph with* $p \geq 3$ *vertices and* $\delta(H) \geq 2$ *.Then* $D_e(G,x) = x^P D_e(H,x)$.

Proof; From the definition of corona product the graph G has $2p$ vertices p pendant edges and p suportating vertices and the degree of any supporting vertices are at least three, that means in G there are p equitable isolated vertices and must be including in any dominating set of G. Also by proposition 2.15, we have $\gamma_e(G) = p + \gamma_e(H)$,therefore for any $1 < i < p \quad d_e(G, p+i) = d_e(H,i)$-------------------(1)

Now, $D_e(G,x) = \sum_{j=p+\gamma_e(H)}^{2p} d_e(G,j)x^j$

$$D_e(G,x) = d_e(G, p + \gamma_e(H)x^{p+\gamma_e(H)} + \cdots \dots \dots \dots + d_e(G,2p)x^{2p}$$, and by (1).

$$D_e(G,x) = D_e\big(H, \gamma_e(H)x^{\gamma_e(H)} + D_e(H, \gamma_e(H) + 1\big)x^{\gamma_e(H)+1} + \cdots \dots \dots \dots + d_e(H,p)x^{2p}$$

$$D_e(G,x) = x^p \sum_{i=\gamma_e(H)}^{p} d_e(H,i)x^i = x^p\, D_e(H,x).$$

Theorem 2.18. *Let H be a graph of $p \geq 3$ vertices and $\delta(H) \geq 2$. Then $D_e\left(Ho\overline{k_m}\right) = x^{mP} D_e(H,x)$.*

Proof: By proposition 2.16 $\gamma_e\left(Ho\overline{k_m}\right) = mp + \gamma_e(H)$. Suppose $G = Ho\overline{k_m}$.Then clearly G has mp pendant vertices all of them are equitable isolated. If A is the set of pendant vertices in G, then for any minimum equitable dominating set $D\ in\ H$ we have $A \cup D$ is minimum equitable dominating set of G. Therefore, $D_e(G, mp+i) = d_e(H,i) where\ \ i = 1,2, \dots .., p$.

$$D_e(G,x) = \sum_{j=\gamma_e(G)}^{2p} d_e(G,j)x^i$$

$$= d_e\big(G, \gamma_e(G)x^{\gamma_e(G)} + d_e(G, \gamma_e(G)+1\big)x^{\gamma_e(G)+1} \dots .. + d_e(G,mp)x^{mp+p}$$

$$= x^{mp} \sum_{j=\gamma_e(H)}^{p} d_e(H,i) = x^{mp} D_e(H,x).$$

Theorem 2.19. *Let G be the spider graph sP_p which construct from the star $k_{1,p}$ by subdivision each edge once as in figure 2. Then $\gamma_e\left(sP_p\right) = p+1\ where\ p \geq 4$.*

Proof: It is clear if $p \geq 4$. Then the degree of v will be more than four but the degrees of the other vertices either one or two as in figure 2.

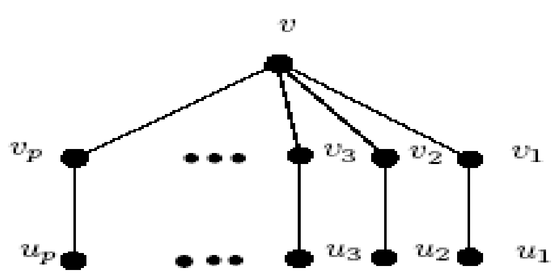

Figure 2.

The head of the spider v is an equitable isolated vertex and shiuld belong to any equitable dominating set of G. Then the minimum equitable dominating set of G will be in the form $\{v\} \cup D$ for some set D, so if $A = \left\{v_{1,} v_{2,}, \dots \dots ., v_p\right\}\ and\ B = \left\{u_{1,} u_{2,}, \dots \dots ., u_p\right\}$ then any minimum equitable dominating set of G will be of the form $\{v\} \cup D$ where $D \subseteq A \cup B$. $\{v\} \cup A\ \ or\ \{v\} \cup B$ are dominating set of size $p+1$,that means $\gamma_e(G) \leq p+1$. Suppose that $\gamma_e(G) < p+1$, then $|\{v\} \cup D| < p+1\ which\ implies\ to |D| < p$ and that is contradiction because in $A \cup B$

every vertex equitable dominates only one vertex so if $|D| < p$ there are some vertices will not dominate. Hence $\gamma_e(sP_p) = p + 1$.

Theorem 2.20. *Let $G \cong sP_p$. Then the number of minimum equitable dominating sets in G is 2^p $for\ any$ $p \geq 4$.*

Proof: Let $G \cong sP_p$where $p \geq 4$ as in figure 2 and let $A = \{v_{1,}v_{2,}, \ldots \ldots., v_p\}$ $and\ B = \{u_{1,}u_{2,}, \ldots \ldots., u_p\}$from Lemma 2.19, we have $\gamma_e(G) = p + 1$ and the head vertex of the spider must be belong to every minimum dominating set as v is an equitable isolated vertex in G.We have to select p vertices from $A \cup B$ we have to select i vertex from A and $p + i$ vertex from B such that if we select v_i $from\ A$ we will select u_j from B such that $i = j$. Then to select zero vertex from A we have to select p vertex from B and if we select one vertex from A we have to select $p - 1$ vertex from B.Clearly to select i vertex from A and $p - 1$ vertex from B there are $\binom{p}{i}$ ways. If we denote the number of minimum equitable dominating sets in G by $\tau_e(G)$, we get

$\tau_e(G) = \binom{p}{0} + \binom{p}{1} + \binom{p}{2} + \cdots \ldots \ldots \ldots \binom{p}{p} = \sum_{i=0}^{p} \binom{p}{i} = 2^p$.Hence, the number of minimum equitable dominating sets in the spider sP_p, where $p \geq 4$ is 2^p .

Theorem 2.21. *For any spider graph sP_p of $2p + 1$ vertices, where $p \geq 4$, $d_e(sP_p, p + 1) = 2^p$.*

Theorem 2.22. *Let G be a friendship graph F_m with $2m + 1$ vertices as in figure 3, where $m \geq 2$. Then (1) $\gamma_e(F_m) = m + 1$*

(2) $d_e(F_m, m + 1) = 2^m$

Proof:

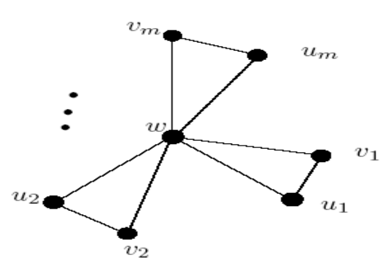

Figure 3.

(1)Clearly the degree of the center vertex w, $\deg(w) \geq 4$ and the other vertices all of degree two. The w is equitable isolated vertex and should be in every dominating set of the graph $G \cong F_m$.Let $D = \{w, v_{1,} v_2, \dots \dots \dots, v_m\}$.Then D is equitable dominating set, that means $\gamma_e(F_m) \leq m + 1$.Now suppose that $\gamma_e(F_m) < m + 1$. Hence, there is equitable dominating set D of size $< m + 1$ which is contradiction since w is an equitable isolated vertex and all the other vertices every vertex has equitable degree one. Hence $\gamma_e(F_m) = m + 1$

(2)To select any minimum dominating set which should be of size $m + 1$ we will select w and one vertex from each triangle. Therefore the number of minimum equitable dominating set in $F_m$is 2^m. Hence $d_e(F_m, m + 1) = 2^m$.

Example 2.23. *Let G be friendship graph F_3 as in figure 4 .To get $D_e(F_3, x)$.*

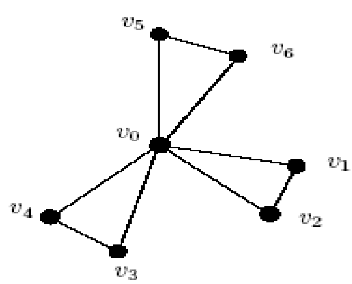

Figure 4.

As in the theorem 2.22 we have $\gamma_e(F_3) = 4$ *and* $d_e(F_{3,} 4) = 2^3 = 8$.

The vertex v_0 must be in every equitable dominating set of F_4. The equitable dominating sets of size four are$\{v_0, v_1, v_3, v_6\}, \{v_0, v_1, v_3, v_5\}, \{v_0, v_1, v_4, v_5\}, \{v_0, v_1, v_4, v_6\}, \{v_0, v_2, v_3, v_6\}$,

$\{v_0, v_2, v_3, v_5\}\{v_0, v_2, v_4, v_6\}, \{v_0, v_2, v_4, v_5\}$, from the figure 4 it is easy to see that any other equitable dominating set of size i whare $s \leq i \leq 7$ is extension for one of the eight equitable dominating set of the size four. Therefore the equitable sets of $F_{3,}$ of size five are $\{v_0, v_1, v_2, v_3, v_5\}, \{v_0, v_1, v_2, v_{4,}, v_5\}, \{v_0, v_1, v_2, v_4, v_6\}, \{v_0, v_1, v_2, v_3, v_6\}, \{v_0, v_1, v_3, v_4, v_6\}$

$\{v_0, v_1, v_3, v_5, v_6\}, \{v_0, v_1, v_3, v_4, v_5\}, \{v_0, v_1, v_4, v_5, v_6\}, \{v_0, v_2, v_3, v_5, v_6\}, \{v_0, v_2, v_4, v_5, v_6\}$

$\{v_0, v_2, v_3, v_4, v_5\}$ and $\{v_0, v_2, v_3, v_4, v_6\}$.

Therefore $d_e(F_3, 5) = 12$.

The equitable dominating sets in F_4 of size 6 . There are only six equitable dominating sets

$\{v_0, v_2, v_{3,} v_4, v_5, v_6\}, \{v_0, v_1, v_3, v_4, v_5, v_6\}, \{v_0, v_1, v_2, v_4, v_5, v_6\}, \{v_0, v_1, v_2, v_3, v_5, v_6\},.$

$\{v_0, v_1, v_2, v_3, v_4, v_6\}, \{v_0, v_1, v_{2,} v_3, v_4, v_5\}$

Thus $d_e(F_3, 6) = 6$, and clearly $d_e(F_3, 7) = 1$.Hence $D_e(F_3, x) = x^7 + 6x^6 + 12x^5 + 8x^4$.

Theorem 2.24. *Let* $\cong F_m$*, where* F_m *is the friendship graph of* $2m + 1$ *vertices and* $m \geq 2$.*Then* $D_e(G, x) = x^{m+1}(x + 2)^m$.

Proof: Let G be a friendship graph F_m with $m + 1$ vertices.Then by proposition 2.22$\gamma_e(G) = m + 1$ *and* $d_e(G, m + 1) = 2^m$. Thus $2^m x^{m+1} + x^{2m+1}$ is containing in the equitable domination polynomial of G. There are 2^m way to select equitable dominating set of size $m + 1$ in G.

Clearly any other equitable dominating set of size $m > m + 1$ should be contains one of the minimum equitable dominating sets of size $m + 1$. To select equitable dominating set in G of size $m + 2$ there are $2^m \binom{m}{1}$ way. But there are repetitions for these equitable dominating set which has same vertices and they are different in one vertex and the different vertices belong to the same triangle that means there are $\frac{1}{2}\binom{m}{1} 2^m$ distinct equitable dominating sets of size $m + 2$ in G. Similarly, every equitable dominating set of size $m + 1$ in G can be extended to equitable dominating set of size $m + 3$ in $\frac{2^m}{4}\binom{m}{2} = 2^{m-2}\binom{m}{2}$ way. Also to construct equitable dominating set of size $m + 4$ there are $\frac{2^m}{8}\binom{m}{3}$ way and so on. In general, there are $\frac{2^m}{2^i}\binom{m}{i}$ way to construct equitable dominating set of size $m + 1 + i$ *where* $1 < i \leq m - 1$.

Therefore, $D_e(G, x) = x^{2m+1} + 2^m x^{m+1} + \frac{1}{2} 2^m \binom{m}{1} x^{m+2} + \frac{1}{4} 2^m \binom{m}{2} x^{m+3} + \cdots \ldots \ldots + \frac{1}{2^{m-1}} 2^m \binom{m}{m-1} x^{2m}$

$$= x^{2m+1} + 2^m x^{m+1} + 2^{m-1}\binom{m}{1} x^{m+2} + 2^{m-2}\binom{m}{2} x^{m+3} + \cdots . +2\binom{m}{m-1} x^{2m}$$

Thus, $D_e(G, x) = x^{m+1}[2^m + x^m + (\sum_{i=1}^{m-1}\binom{m}{i} 2^{m-i} x^i)]$

$$= x^{m+1}[\, 2^m + x^m + \left(\sum_{i=1}^{m-1}\binom{m}{i} 2^{m-i} x^i\right) - (2^m + x^m)]$$

$$= x^{m+1}[\, \sum_{i=0}^{m}\binom{m}{i} 2^{m-i} x^i \,]$$

$$= x^{m+1}(x + 2)^m$$

Hence $D_e(F_{m,}, x) = x^{m+1}(x+2)^m$ *where* $m \geq 2$.

Corollary 2.25. *For any friendship graph F_m with $m \geq 2$ there are only two equitable domination roots which is zero and -2.*

Definition: *The m-book graph is defined as the Cartesian product $k_{1,m-1} \square k_2$ as in figure 5.*

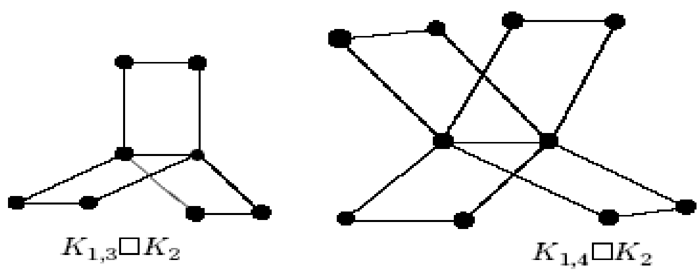

Figure 5.

Theorem 2.26. *Let G be the book graph $B_m \cong k_{1,m}\square\ k_2$.Then$D_e(G,x) = x^{m+2}(x+2)^m$ where $m \geq 3$.*

Proof: Let $G \cong B_m$ as in figure 6.

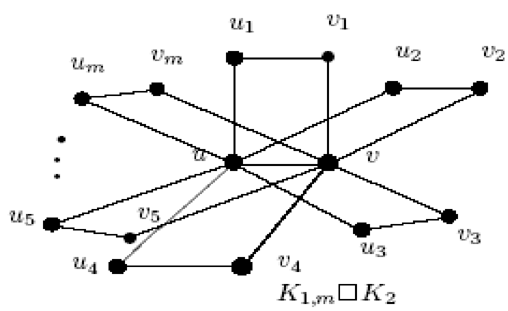

Figure 6.

From the figure clearly, the vertices of the book graph B_m, u *and* v are equitable isolated vertices and should be in any equitable dominating se of G and it is easy to see that $\gamma_e(G) = m+2$. To get the number of minimum equitable dominating set in $G \cong B_m$ the two vertices u *and* v must be in the minimum equitable dominating set and we have to select one vertex from every set $\{v_i, u_j \quad i \in \{1,2,\dots\dots,m\}$. That means we have 2^m ways to construct minimum equitable dominating set of size $m+2$.Therfore $d_e(G,m+2)=2^m$ and clearly $d_e(G,2m+2)=1$. If D is any equitable dominating set of size $i\ where\ i>m+2$ is extended set for some minimal equitable dominating set.In other meaning any equitable dominating set in G contains some minimum equitable dominating set. Now, to count the number of equitable dominating set of size $m+3$, every equitable dominating set of size $m+2$ can be extended to equitable dominating set in $\binom{m}{1}$ way.That means we can construct equitable dominating set of size $m+3$ in $m2^m$ way , but any two equitable dominating sets of size $m+2$ which they are identical in all the vertices except one vertex will extend to the same equitable dominating set of size $m+3$ .Therfore, there are $\frac{m}{2}2^m = m2^{m-1}$ way to get equitable dominating set of size $m+3$. Hence $d_e(G,m+3)=m2^{m-1}$. Now to count the equitable dominating sets in G of size $m+4$, we have two methods. either by extend the equitable dominating sets of size $m+3$ to equitable dominating set of size $m+4$ by adding one vertex or by extend the equitable dominating sets of size $m+2$ to size $m+4$ by adding two vertices similarly we have $\frac{1}{4}2^M\binom{M}{2}$ way to get distinct equitable dominating of size $m+4$ and so on , we have

$d_e(G,m+3)=2^m\binom{m}{1}.\frac{1}{2}$.

$d_e(G,m+4)=2^m\binom{m}{2}.\frac{1}{4}$.

$d_e(G,m+5)=2^m\binom{m}{3}.\frac{1}{8}$.

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. $d_e(G,2m+1)=2^m\binom{m}{M-1}.\frac{1}{2^{m-1}}$.

Therefore,

$$D_e(G,x)=x^{2m+2}+2^m x^{M+2}+2^{m-1}\binom{m}{1}x^{M+3}+2^{m-2}\binom{m}{2}x^{M+4}+2^{m-3}\binom{m}{3}x^{M+5} + 2^{m+5}\binom{m}{4}x^{M+6}+\cdots\ldots\ldots\ldots.+2\binom{m}{M-1}x^{2m+1}$$

$$=x^{2m+2}+2^m x^{M+2}+x^{M+2}[2^{m-1}\binom{m}{1}x+2^{m-2}\binom{m}{2}x^2+\cdots\ldots..+2\binom{m}{M-1}x^{m-1}]$$

$= x^{2m+2} + 2^m x^{M+2} + x^{M+2}[\left(\sum_{i=0}^{m} 2^{M-i} \binom{m}{i}^i x\right) - (2^m + x^m)]$

$= x^{2m+2}(2+x)^m$.

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