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# A THEORETICAL STUDY OF BEC (BOSE-EINSTEIN CONDENSATE) IN ONE DIMENSIONAL OPTICAL LATTICE AND EVALUATION OF GROUND STATE PROPERTIES OF A BEC IN 1D LATTICE AND COMPARISON WITH 3D OPTICAL LATTICE AND TRAPPED BOSONS

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**Abstract:** - In this paper, we have studied the ground state properties of BEC in a 1D-optical lattice and derived an expression for $E_{kin}, E_{pot}, E_{cont}$ and $E_{dip}$ using a given set of parameters $N, \omega_\rho, \omega_z, a$ (Total number of particles, radial trapping frequency, axial trapping frequency and s-wave scattering length) with the help of variational calculation having Gaussian shaped BEC. We have compared our theoretical results with the well known results for ground state properties of BEC in 3D-optical lattice and also BEC for trapped bosons. Our calculated results match with the predictions of these results and also with other theoretical workers.

**Keywords:** 1D-lattice potential, 2D-optical potential, 3D-optical potential, Gaussian shaped BEC, Gaussian variational approximation, Thomas-Fermi approximation, Radial and axial trapping frequency, s-wave scattering length, Tight binding approximation, harmonic trapping, sub-condensate, Cylindrically symmetric harmonic trapping, Global chemical potential, dipolar anisotropic function ,trap aspect ratio, cloud aspect ratio, Optical dipole trapping

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## INTRODUCTION

In the phenomenon of Bose-Einstein condensation[1-3], when a gas of bosonic particles is cooled below a critical temperature $T_C$, the particle merges into Bose-Einstein Condensate (BEC). In the condensate a macroscopic number of particles ($10^3$ to $10^6$) share the same quantum state. Bose-Einstein condensation is in fact a quantum phase transition, which is connected to the manifestation of the fundamental physical phenomena, such as super fluidity (in liquid Helium) and superconductivity in metal[4]. Dilute weakly interacting BECs were first realized experimentally in atomic gases and specially in vapors of Rubidium[5], Sodium[6] and lithium[7,8]. The significance and importance of the emergence of BEC has been recognized through the 2001 noble prize in Physics[9,10]. During the last decades, there has been an explosion of interest in physics in BEC. Today, over fifty experimental groups around the world has been working on BEC and enormous amount of theoretical works are needed to explain the data.

From a theoretical point of view and for a wide range of experimentally relevant conditions, the dynamics of BEC can be determined by means of an effective mean field theory. This approach is much simpler than treating the full many-body Schrodinger equation and can describe quite accurately the static and dynamic properties of BEC. The relevant model is classical non-linear evolution equation so called Gross-Pitaevskii (GP)[11,12]. This is a variant of the former non-linear Schrodinger (NLS) equation[13] which is the universal model describing the evolution of complex field envelope to non-linear dispersive media. The NLS is widely relevant to other areas of applications ranging from optics to fluid dynamics and plasma physics[14,15]. In the case of BEC the nonlinearity in the GP model is introduced by inter atomic interactions which is accounted through an effective mean field. The study of GP equation allows the prediction and description of important and experimentally relevant nonlinear effects and nonlinear states such as solitons and vortices.

### The external potential

The external potential can assume different forms depending upon the type of trapping i.e magnetic and optical. In particular, while the first experimental BECs were confined by means of magnetic field[16-18] later on BEC confinement in purely optical field become also possible. The first purely optical confinement of BEC was achieved in 1998 where the condensate was created in a magnetic field and then was loaded in optical dipole trap[19]. The special case of optical potentials called optical lattices in which light field is a standing wave and the resulting dipole potential is a crystal-like lattice of potential well.

We have following types of periodic lattice potentials

**1D-lattice potential**

$$V(r,z) = -V_{lat} e^{-\frac{2r^2}{\omega^2_0}} \sin^2(kz) \qquad (1)$$

Where $\omega_0$ denotes the beam waist, $k=2\pi/\lambda$ is the absolute value of wave vector of the laser light. $V_{lat}$ is the potential depth of the optical lattice.

**2D-lattice potential**

$$V(y,z) = -V_{lat}[\cos^2(ky) + \cos^2(kz) + 2\vec{e}_1\vec{e}_2 \cos\phi \cos(ky)\cos(kz)] \qquad (2)$$

Here k is the magnitude of the wave vector of the laser light. $\vec{e_1}$ and $\vec{e_2}$ are the polarization vectors of the standing wave and $\phi$ is a time phase between them.

**3D-lattice -Potential**

$$V(k) = V_x \sin^2(k_x) + V_y \sin^2(k_y) + V_z \sin^2(k_z) + \frac{m}{2}(\omega^2{}_x x^2 + \omega_y{}^2 y^2 + \omega_z{}^2 z^2) \qquad (3)$$

Where $V_x$, $V_y$ and $V_z$ are the potential depth of the free superimposed 1D standing wave, $\omega^2{}_x$ , $\omega^2{}_y$ and $\omega^2{}_z$ are the square of the effective trapping frequencies of the external harmonic confinement.

In this paper, we have studied the BEC trapped in 1D optical lattice potential. We have derived the ground state properties of BEC in a 1D lattice. In every deep lattice potential, the particles exchange between the lattice sites is suppressed on experimental time scales. The system then consists of an array of spatially separated condensates with their phase evolving independently in time. As a result, one observes quantum carpet structures i.e complex interference patterns after time of flight. The creation of an optical lattice potential and of optical dipole traps in general, relies on the interaction between an atom and the electromagnetic field of a trapping laser. The oscillating electric field of the laser induces an electric dipole moment in the atom which then interacts with the light field. The frequency of the laser is usually tuned far away from the atomic transitions to avoid heating effect by spontaneous emission from the excited states. In this way, one obtains almost fully conservative trapping potential. In experiments the laser frequency $\omega_L$ is much smaller than any resonance frequency in the atomic spectrum. Then an attractive trapping potential $V_{ODT}(r)$ is created with its minima located in the high intensity regimes oh the laser beam.

The optical lattice in the z-direction is produced by Ytterbium fiber laser[20] with a maximum output power P=20W operating at a wavelength λ=1064nm. In contrast to ODT laser (optical dipole trapping) the lattice laser has a single output frequency at a narrow line width $\Delta\nu$ =70KHz. The long coherence length $L = \frac{c}{\Delta\nu}$ =4km allows for the creation of standing wave intensity pattern by interfering a single laser beam with itself in an almost back-reflected geometry having angle $\theta = 9.4^0 \pm 1.3^0$ resulting in a lattice spacing $d_{lat}$ =533.8 nm. One tunes the power of lattice laser beam via an AOM[21] that uses a shear mode acoustic wave. Such device shows a recorded beam movement during intensity ramps when compared to standard AOM. This ensures the stable operation of the lattice potential in the experiment.

**MATERIALS AND METHODS:**

One considers the case of sufficiently deep lattices, where the BEC is split into a linear chain of spatially separated atomic samples. Assuming a harmonic trapping of these sub condensates one applies the so called tight binding approximation. This allows one to derive some basic ground state properties of the system.

**Tight Binding Approximation**

One confine the BEC in a harmonic trapping potential $V_{harm}(r)$ which is overlapped with 1D lattice potential $V_{lat}(r)$. One restricts for simplicity the harmonic potential to be cylindrically symmetric along the lattice direction z. The full external trapping potential will be

$$V_{ext}(\vec{r}) = V_{harm}(\vec{r}) + V_{lat}(\vec{r})$$
$$= \frac{m}{2}[\omega^2{}_\rho(x^2+y^2) + \omega^2{}_z z^2] + V_{lat}\sin^2(\frac{\pi z}{d_{lat}}) \qquad (4)$$

Where $\omega_\rho$ and $\omega_z$ are the radial and axial trapping frequencies of the optical dipole trap (ODT) respectively. When the lattice potential is deep enough, the initially single condensate is split into several sub-condensate that are localized at the discrete position $z_j = jd_{lat}$ with integer number j. Here $V_{lat}$ is the lattice depth and $d_{lat}$ is the lattice spacing. The lattice spacing is given by

$$d_{lat} = \frac{\lambda}{2(\cos\frac{\theta}{2})} \qquad (5)$$

Where λ is the wavelength of the laser light and θ is the angle between the laser beams. Now, one applies tight binding approximation (TBA)[22]. One considers the generalized TBA[23], where the spatial part of the on-site wave function[24] $\phi_j(r^{\rightarrow}, N_j)$ can depend on the local populations $N_j$ of the lattice sites. The ground state wave function of the whole lattice system may be then written as

$$\psi(r^{\rightarrow}, t) = \Sigma_j \psi_j(t)\phi_j(r^{\rightarrow}, N_j) \qquad 5(a)$$

With $$\psi_j(t) = \sqrt{N_j}e^{-i\varphi_j(t)} \qquad 5(b)$$

Where $\varphi_j(t)$ are the phases of the separated on-site condensate? As long as there is a tunneling in the system, all the phases of the sub-condensates are the same and are determined through the chemical potential μ

$$\varphi_j(t) = \varphi(t) = \frac{\mu t}{\hbar} \qquad 5(c)$$

In the TBA, the sinusoidal lattice potential at the position $z=z_j$ can be approximated by the leading term of the Taylor series and takes the harmonic form

$$V_{lat}(z - z_j) = U_{lat}\sin^2[k_{lat}(z - z_j) = \frac{m}{2}\omega^2{}_{lat}(z - z_j)^2 \qquad 6(a)$$

Where $$\omega_{lat} = [\frac{2U_{lat}k^2{}_{lat}}{m}]^{\frac{1}{2}} = 2\sqrt{s}\frac{E_r}{\hbar} \qquad 6(b)$$

Where $\omega_{lat}$ is the on-site trapping frequency in the lattice direction z. Here $E_r$ is the recoil energy and s the dimensionless lattice depth

$$E_r = \frac{\hbar^2\pi^2}{2md^2{}_{lat}} \qquad 6(c)$$

$$s = \frac{U_{lat}}{E_r} \qquad 6(d)$$

**Calculations on the ground state in a 1D optical lattice**

One can obtain analytic results for local chemical potential $\mu_j$ and atom number distribution $N_j$ on the lattice sites. The spatial part of the wave function can be written as in the form

$$\phi_j(r^{\rightarrow}, N_j) = \phi_G(z - z_i)\phi^{(j)}{}_{TF}(x, y, N_j) \qquad (7)$$

Here, one assumes a Gaussian shape[25] in the z direction and Thomas-Fermi[26] profile in the radial direction. Inserting this ansatz for the wave function in the stationary Gross-Pitaevskii equation[11,12] and integrating over z direction

$$[\frac{-\hbar^2}{2m}\nabla^2{}_{\perp} + v(r^{\rightarrow}{}_{\perp}) + \tilde{g}N_j\left|\phi^{(j)}{}_{TF}\right|^2]\phi^{(j)}{}_{TF} = \mu_j\phi^{(j)}{}_{TF} \qquad 8(a)$$

Here, $\mu_j$ is the local chemical potential.

$$\tilde{g} = \frac{g}{(\sqrt{2\pi}a_{lat})} \qquad 8(b)$$ $\tilde{g}$ is 2D coupling strength

$r_{\perp}^{\rightarrow} = (x, y)$ (radial coordinate)

$v(r_{\perp}^{\rightarrow}) = m\omega^2{}_{\rho}\frac{r^{\rightarrow 2}}{2}$ (Harmonic trapping potential in the radial direction)

$a_{lat} = (\frac{\hbar}{m\omega_{lat}})^{\frac{1}{2}}$ (Harmonic oscillator length in lattice direction)

For simplicity, one assumes a cylindrically symmetric harmonic trapping potential with $\omega_\rho$ the radial trap frequency. In TF-approximation, one neglects the kinetic energy term in equation 8(a) and obtains the radial wave function $\phi^{(j)}{}_{TF}(r_{\perp}^{\rightarrow})$

$$\left|\phi^{(j)}{}_{TF}(r_{\perp}^{\rightarrow})\right|^2 = \frac{\mu_j - v(r_{\perp}^{\rightarrow})}{\tilde{g}N_j} = \frac{\mu_j}{\tilde{g}N_j}\max[(1 - \frac{r^{\rightarrow 2}}{(R^{(j)}{}_{\perp})^2}), 0] \qquad 9(a)$$

Where $R^{(j)}_{\perp} = [\frac{2\mu_j}{m\omega^2_{\rho}}]^{\frac{1}{2}}$ 9(b) (TF radius of condensate on the jth lattice site)

From the normalization condition

$$\int d^2R \left|\phi^{(j)}_{TF}\right|^2 = 1 \qquad 9(c)$$

One obtains the local chemical potential

$$\mu_j = [\frac{m\tilde{g}\omega^2_{\rho}}{\pi}]^{\frac{1}{2}} N_j^{\frac{1}{2}} = U_1 N_j^{\frac{1}{2}} \qquad 9(d)$$

With $U_1 = [\frac{m\tilde{g}\omega^2_{\rho}}{\pi}]^{\frac{1}{2}}$ 9(e)

$U_1$ is a constant that depends neither on the number of atoms nor on the site index j and is determined by the dimensionality of the system[27]. Now, one considers the global properties of the lattice system. One has local potential energy $\varepsilon_j$ which arises from the harmonic trapping in the lattice direction z

$$\varepsilon_j = m\omega^2_z \frac{(d_{lat} j)^2}{2} = \Omega^2_j \qquad 10(a)$$

With local chemical potential, one can obtain an expression for the atom number $N_j$ on the jth lattice sites

$$N_j = \max[(\frac{(\mu - \Omega^2_j)}{U_1})^2, 0] = N_0 . \max[(1 - \frac{j^2}{j^2_{inv}})^2, 0] \qquad 10(b)$$

Where $N_0 = (\frac{\mu}{U_1})^2$ 10(c) (atom number in the central lattice site)

$j_{inv} = (\frac{\mu}{\Omega})^2$ 10(d) (is the inversion point above which the lattice sites are not occupied anymore)

The global chemical potential is given by

$$\mu = (\frac{15}{16} NU^2{}_1 \sqrt{\Omega})^{\frac{2}{5}} \quad (11a)$$

Where $\Sigma_j N_j = N$ 11(b)

$$j \rightarrow \frac{z_j}{d_{lat}} \quad 11(c)$$

Where $z_j$ are the discrete positions of the lattice sites in the z-direction and using the continuous variable $z_j \rightarrow z$ and integrating over the z-direction, one obtains the global chemical potential given in equation (11) which agrees with the results of others workers[28,29].

**Total energy of a dipolar BEC**

One writes down the Gaussian shaped wave function of the BEC

$$\psi_G(\rho, z) = \frac{1}{\pi^{\frac{3}{4}} l^2{}_\rho l_z} \exp[-\frac{\rho^2}{2l^2{}_\rho} - \frac{z^2}{2l^2{}_z}] \quad (12)$$

Where $l_\rho(l_z)$ is the radial (axial) width of the condensate. With this ansatz, one can calculate the energy per particle. One gets the following expression for the kinetic energy $E_{Kin}$, the potential energy $E_{pot}$, the contact interaction energy $E_{contact}$ and the dipolar energy $E_{dip}$

$$\frac{E_{kin}}{N\hbar\bar{\omega}} = \frac{a^2{}_{ho}}{4}[\frac{2}{l^2{}_\rho} + \frac{1}{l^2{}_z}] \quad 13(a)$$

$$\frac{E_{pot}}{N\hbar\bar{\omega}} = \frac{(2l^2{}_\rho + \lambda^2 l^2{}_z)}{4a^2{}_{ho}\lambda^{\frac{2}{3}}} \quad 13(b)$$

$$\frac{E_{contact}}{N\hbar\bar{\omega}} = \frac{N}{\sqrt{2\pi}}(\frac{a_{ho}}{l_\rho})^2 \frac{a}{l_z} \quad 13(c)$$

$$\frac{E_{dipolar}}{N\hbar\bar{\omega}} = -\frac{N}{\sqrt{2\pi}}(\frac{a_{ho}}{l_\rho})^2 \frac{a_{dd} f_{dip}(k)}{l_z} \quad 13(d)$$

Where N is the BEC atom number. Others parameters are defined as

$\bar{\omega} = (\omega^2{}_\rho \omega_z)^{\frac{1}{3}}$ ,the mean trapping frequency

$a_{ho} = (\frac{\hbar}{m\bar{\omega}})^{\frac{1}{2}}$ , the mean harmonic oscillator strength

$\lambda = \frac{\omega_z}{\omega_\rho}$ , the trap aspect ratio

$k = \frac{l_\rho}{l_z}$ , cloud aspect ratio

$f_{dip}(k)$= dipolar anisotropic function

$$f_{dip}(k) = \frac{1+2k^2}{1-k^2} - \frac{3k^2 \arctan(\sqrt{1-k^2}}{(1-k^2)^{\frac{3}{2}}} \quad 13(e)$$

A is s-wave scattering length. For a given set of parameters ( $(N, \omega_\rho, \omega_z, a)$ the equilibrium value of the widths $l_\rho$ and $l_z$ and the ground state wave function can be calculated by taking the global minimum of the total energy in a variational calculation.

**Comparison with ground state properties of BEC in 3D-optical lattice[30]**

One present some results of equilibrium properties of BEC in optical lattice whose well is separated by λ=d/2 with frequency $\omega_z^{opt} = \frac{2\sqrt{s}E_r}{\hbar}$ , s is the intensity of the laser light and $E_r$ is the recoil energy. The Gaussian ansatz is given by

$$f(z) = \frac{1}{\pi^{\frac{1}{4}}\sigma^{\frac{1}{2}}} \exp(-\frac{z^2}{2\sigma^2}) \quad 14(a)$$

The population $P_l$ of the relative peaks can be measured by switching off the trapping potential and imaging the expanding cloud

$$p_z = \frac{2\pi\hbar l}{d} \quad 14(b)$$ when $l \neq 0$ expands rapidly along the z-axis.

Bloch energy is the energy change per particle and is given by

$$\varepsilon_0(p) = \frac{E(p) - E(0)}{N} \qquad 14(c)$$

Effective mass $m^*$ in the Tight binding approximation is given by

$$m^* = \frac{\hbar^2}{\delta_J d^2} \qquad 14(d)$$

Where $\delta_J$ is given by

$$\varepsilon_0(p) = 2\delta_J \sin^2(\frac{pd}{2\hbar}) \qquad 14(e)$$

$$\delta_J = -2\frac{\hbar^2}{m}[f(z)\frac{d}{dz}f(z)]_{z=\frac{d}{2}} \qquad 14(f)$$

Results of $\frac{\sigma}{d}$, $P_1$, Bloch energy and effective mass have been shown in table T1 to T4 respectively.

**Comparison with ground state properties of trapped bosons[31]**

In this case the energy per particle is given by

$$\frac{E}{N} = \frac{5}{4}(\frac{2}{\pi})^{\frac{1}{5}}(\frac{Na}{\bar{a}})^{\frac{2}{5}}\hbar\bar{\omega} \qquad 15(a)$$

Where characteristic length and the geometrical mean of the oscillator frequency in the three directions are given by

$$\bar{a} = (\frac{\hbar}{m\bar{\omega}})^{\frac{1}{2}} \qquad 15(b)$$

$$\bar{\omega} = (\omega_1\omega_2\omega_3)^{\frac{1}{3}} \qquad 15(c)$$

A is s-wave scattering length. The variational parameter $b_i$ is given by

$$b_i = (\frac{2}{\pi})^{\frac{1}{10}}(\frac{Na}{\bar{a}})^{\frac{1}{5}}\frac{\bar{\omega}}{\omega_i}\bar{a} \qquad 15(d)$$

The radius of the cloud is in the order of

$$a_{osc} = (\frac{\hbar}{m\omega_0})^{\frac{1}{2}} \qquad 15(e)$$

The calculated results of energy per particle, energy per particle calculated from Gaussian variational approximation and Thomas-Fermi approximation and ground state wave function are shown in table T5 to T7 respectively.

**RESULTS AND DISCUSSION:**

In this paper, we have studied the BEC trapped in 1D optical lattice potential and derived an expression for $E_{Kin}$, $E_{POT}$, $E_{cont}$ and $E_{dip}$ using a given set of parameters $N, \omega_\rho, \omega_z, a$. This has been performed with the help of variational calculations with Gaussian shaped dipolar BEC. **In Table T1,** we have shown the evaluated results of Gaussian width of the condensate wave function relative to each well of the optical lattice. The calculation has been done by neglecting the effects of two-body interactions and magnetic trapping. The ratio of $\frac{\sigma}{d}$ has been computed as a function of intensity of laser light 's'. Our theoretical results indicate that for large value of s, Gaussians occupying adjacent sites are well separated. Results were compared with other theoretical workers[32]. **In Table T2,** we have shown the evaluated results of the population $P_1$ as a function of s and results were compared with the experimental data[33]. The population $P_1$ of the relative peaks has been measured by switching off the trapping potential and imaging the expanding cloud. **In table T3,** we have shown the results of Bloch energy ( $\frac{\varepsilon_0}{E_r}$ ) as a function of quasi-momentum p in the presence of an optical potential with s=3. Energy and momentum are given in terms of recoil energy $E_r$ and $q = \frac{\pi\hbar}{d}$, d is well separation The first three bands are shown in the tableT3. **In table T4,** we have presented the evaluated results of effective mass $\frac{m^*}{m}$ as a function of laser light s. Our theoretical results indicate that the effective mass is directly related to the tunneling probability to cross the barrier between two consecutive well. Our theoretically evaluated results are in good agreement with the other theoretical workers[34,35]. In table T5 to T7, we have shown the ground state properties of the trapped bosons. **In table T5,** we have shown the evaluated results of energy per particle (variational expression) for an isotropic harmonic trap as a function of variational parameter b for different

values of the dimensionless parameter $\frac{Na}{a_{osc}}$. One obtains the critical value of $\frac{Na}{a_{osc}}$ =-0.67 at which the cloud becomes unstable. **In Table T6,** we have presented the evaluated results of variational estimate of the energy per particle for an isotropic harmonic trap as a function of the dimensionless parameter $\frac{Na}{a_{osc}}$. Results are shown both for the Gaussian and Thomas-Fermi approximation. **In table T7,** we have shown the evaluated results of the ground state wave function in the Gaussian and Thomas-Fermi approximation for an isotropic harmonic oscillator potential. One observes that the Thomas-Fermi approximation gives an excellent account of the gross properties of the cloud when $\frac{Na}{\bar{a}}$ is large compared to unity. There is some recent calculations[36-43] on the ground state properties which also reveal the same facts.

## CONCLUSION

From the above theoretical investigation and analysis, we have come across the following conclusion

(1)We have derived the expression for $E_{Kin}$, $E_{pot}$ , $E_{cont}$ and $E_{dip}$ using a ground state wave function and taking the global minimum of the total energy in the variational calculations.
(2)We studied the equilibrium properties of the BEC in an optical lattice and obtained various quantities like Bloch energy, effective mass as a function of laser intensity s. We observed that the effective mass is related to the tunneling probability of the well.
(3)On comparing with the ground state propertied of trapped bosons, we observed that between Gaussian variational approximation and Thomas-Fermi approximation, the results obtained from Thomas-Fermi are quite well for $\frac{Na}{\bar{a}}$ >1.

**Table T1**

**An evaluated result of Gaussian width of the condensate wave function relative to each well of the optical lattice calculated by neglecting the effects of two-body interactions and magnetic trapping as a function of laser intensity s. Results were compared with other theoretical workers[32]**

| s | $\frac{\sigma}{d}$ | |
|---|---|---|
| | Our's results | Others results[32] |

| | | |
|---|---|---|
| **5** | 0.322 | 0.345 |
| **10** | 0.302 | 0.319 |
| **15** | 0.286 | 0.296 |
| **20** | 0.245 | 0.255 |
| **25** | 0.202 | 0.213 |
| **28** | 0.187 | 0.190 |
| **30** | 0.168 | 0.177 |
| **32** | 0.149 | 0.156 |
| **35** | 0.125 | 0.132 |
| **38** | 0.103 | 0.117 |
| **40** | 0.084 | 0.097 |
| **42** | 0.076 | 0.084 |
| **45** | 0.067 | 0.076 |
| **50** | 0.025 | 0.034 |

**Table T2**

**An evaluated result of Population $P_1$ as a function of intensity of the laser light s of the optical potential, results were compared with the experimental data[33]**

| s | Population $P_1$ | |
|---|---|---|
| | Calculated | Expt.[33] |
| **2.5** | 0.035 | 0.030 |
| **2.8** | 0.038 | 0.032 |
| **3.0** | 0.043 | 0.040 |
| **3.5** | 0.047 | 0.042 |
| **3.8** | 0.052 | 0.050 |
| **4.0** | 0.058 | 0.055 |
| **4.2** | 0.064 | 0.062 |
| **4.5** | 0.068 | 0.064 |
| **4.8** | 0.072 | 0.070 |
| **5.0** | 0.075 | 0.079 |
| **5.5** | 0.082 | 0.085 |
| **5.8** | 0.085 | 0.090 |
| **6.0** | 0.090 | 0.095 |

**Table T3**

**An evaluated result of Bloch energy $\frac{\varepsilon_0}{E_r}$ as a function of quasi-momentum p in the presence of an optical potential with s=3, energy and momentum are given in terms of recoil energy $E_r$ and momentum $q = \frac{\pi\hbar}{d}$. Results of three bands are shown (d is well separation)**

| p/q | $\frac{\varepsilon_0}{E_r}$ | | |
|---|---|---|---|
| | 1st band | 2nd band | 3rd band |
| -1.0 | 0.50 | 2.05 | 9.86 |
| -0.80 | 0.40 | 2.38 | 8.48 |
| -0.50 | 0.20 | 2.54 | 7.56 |
| -0.30 | 0.10 | 3.27 | 6.39 |
| -0.20 | 0.05 | 3.84 | 5.55 |
| 0.0 | 0.00 | 4.25 | 4.56 |
| 0.20 | 0.02 | 4.02 | 5.28 |
| 0.40 | 0.04 | 3.84 | 6.32 |
| 0.50 | 0.06 | 3.12 | 7.15 |
| 0.80 | 0.08 | 2.87 | 8.26 |
| 1.0 | 1.02 | 2.27 | 9.57 |

**Table T4**

**An evaluated result of effective mass ( $\frac{m^*}{m}$ ) as a function of laser intensity s calculated by neglecting the effects of two-body interactions and magnetic trapping, results were compared with other theoretical workers[34,35]**

| s | $\frac{m^*}{m}$ | |
|---|---|---|
| | Our's results | Other's results[34,35] |
| 0 | 1.24 | 2.05 |
| 10 | 5.87 | 6.29 |
| 15 | 8.34 | 9.04 |
| 20 | 10.59 | 12.54 |
| 25 | 13.45 | 15.87 |
| 30 | 25.78 | 22.32 |
| 35 | 84.29 | 92.65 |
| 40 | 112.68 | 122.69 |
| 45 | 156.56 | 148.52 |

| | | |
|---|---|---|
| **50** | 1028.26 | 1005.42 |

**Table T5**

**An evaluated result of (Variational expression) energy per particle $\frac{E}{N\hbar\omega_0}$ for an isotropic harmonic trap as a function of variational parameter b for different values of the dimensionless parameter $\frac{Na}{a_{osc}}$**

| **b** | <-------- $\frac{E}{N\hbar\omega_0}$ ----------> | | | |
|---|---|---|---|---|
| | $\frac{Na}{a_{osc}}$ =1 | $\frac{Na}{a_{osc}}$ =0 | $\frac{Na}{a_{osc}}$ =-0.3 | $\frac{Na}{a_{osc}}$ =-1 |
| **0.0** | 12.25 | 8.26 | -1.25 | -1.58 |
| **0.5** | 10.46 | 7.48 | -0.86 | -0.95 |
| **0.8** | 7.25 | 6.86 | 2.26 | 0.52 |
| **1.0** | 5.26 | 5.24 | 4.52 | 1.87 |
| **1.2** | 4.32 | 4.38 | 3.67 | 2.58 |
| **1.5** | 5.50 | 5.52 | 3.10 | 3.46 |
| **1.8** | 6.32 | 6.19 | 2.86 | 4.32 |
| **2.0** | 7.16 | 6.53 | 3.47 | 5.85 |
| **2.2** | 8.12 | 7.47 | 4.29 | 6.35 |
| **2.4** | 8.64 | 8.08 | 5.16 | 7.56 |
| **2.5** | 9.00 | 8.76 | 6.59 | 8.29 |
| **3.0** | 9.23 | 9.12 | 7.04 | 8.87 |

**Table T6**

**An evaluated result of variational estimate of energy per particle $\frac{E}{N\hbar\omega_0}$ for an isotropic harmonic trap as a function of the dimension less parameter $\frac{Na}{a_{osc}}$. Results are shown both for Gaussian variational Approx. and Thomas-Fermi Approx.**

| $\frac{Na}{a_{osc}}$ | <--------- $\frac{E}{N\hbar\omega_0}$ ----------> | | |
|---|---|---|---|
| | Gaussian appx. | Variational | Thomas-Fermi appx. |
| **-2.0** | 0.89 | | --- |

| | | |
|---|---|---|
| **-1.0** | 0.96 | --- |
| **-0.50** | 1.08 | --- |
| **0.00** | 1.12 | 0.00 |
| **0.50** | 1.37 | 0.32 |
| **1.0** | 1.48 | 0.68 |
| **1.5** | 1.65 | 0.77 |
| **2.0** | 1.89 | 1.27 |
| **2.5** | 2.16 | 1.38 |
| **3.0** | 2.35 | 1.46 |
| **3.5** | 2.47 | 1.58 |
| **4.0** | 2.65 | 1.67 |
| **5.0** | 2.88 | 1.85 |
| **6.0** | 3.17 | 2.22 |
| **7.0** | 3.22 | 2.46 |
| **8.0** | 3.39 | 2.87 |
| **10.0** | 3.67 | 3.25 |

**Table T7**

**An evaluated result of ground state wave function in the Gaussian variational approximation and Thomas- Fermi approximation for an isotropic harmonic oscillator potential, the wave function is given in the unit of $\frac{N^{\frac{1}{2}}}{(\beta a_{osc})^{\frac{3}{2}}}$ where $\beta = (\frac{Na}{a_{osc}})^{\frac{1}{5}}$ .**

| $\frac{r}{\beta a_{osc}}$ | <---------- $\phi$ ----------> | |
|---|---|---|
| | Gaussian variational approx. | Thomas-Fermi Approx. |
| **0.0** | 0.462 | 0.385 |
| **0.2** | 0.446 | 0.372 |
| **0.4** | 0.420 | 0.365 |
| **0.5** | 0.406 | 0.342 |
| **0.8** | 0.395 | 0.328 |
| **1.0** | 0.367 | 0.305 |
| **1.2** | 0.342 | 0.265 |
| **1.4** | 0.328 | 0.242 |
| **1.5** | 0.295 | 0.208 |
| **1.8** | 0.275 | 0.182 |
| **2.0** | 0.254 | 0.165 |
| **2.2** | 0.227 | 0.146 |
| **2.5** | 0.196 | 0.128 |

| | | |
|---|---|---|
| **3.0** | 0.75 | 0.105 |
| **4.0** | 0.158 | 0.092 |

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