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A THEORETICAL STUDY OF TRAPPED ATOMIC POLARITON IN A BICONICAL WAVEGUIDE CAVITY AND EVALUATION OF CRITICAL TEMPERATURE T_C AS A FUNCTION OF LB POLARITON NUMBER

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Abstract: - Using the theoretical formalism of **I. Yu. Chestnov etal (2012),** we have studied BEC for trapped atomic polaritons in a biconical waveguide cavity. We have theoretically evaluated density of states ρ(ε) as a function of energy using two potential (a) an exact trapping potential U(z) and (b) power law potential $U(z)^v$ for different trapping power parameter ν. We have also evaluated critical temperature T_C of trapped atomic polaritons. Our estimated values of T_C is quite large (T_C>530K) which shows that trapped polariton behaves photon-like character due to their low effective mass. Our theoretically evaluated results are in good agreement with other theoretical workers.

Keywords: Atomic polaritons, Biconical waveguide cavity, 2D gas of bosonic particles, Kosterlitz-Thouless (K-T) phase transition, nanosecond domain, Coupled atom-light (dressed) states, Optical collisions, Thermal quasi-equilibrium, Optical pumping effects, Trapping power parameter.

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INTRODUCTION

The investigation of quantum and statistical properties of Bose-gases in low and especially in 1D has evoked indefatigable interest in atomic optics and condensed matter physics for last few decades[1-9]. In particular at finite temperature for 1D and 2D ideal Bose gases a true BEC can only be achieved in the presence of suitable trapping potential[4]. The critical temperature for the phase transition depends on the shape of the trapping potential which is usually harmonic in practice[3,4]. Low dimensional systems have been studied using atoms in highly deformed traps[10], where the effects of dimension reduction become important[11].Alternative systems are bosonic quasi particles where light and matter are coupled in a coherent way[12]. Here quantum and statistical properties of light Bose-quasi particles like excitons[13], magnons[14] and polaritons[15] have been considered. For example excitons-polaritons occurring in quantum well structures placed in micro-cavities can be treated as a 2D gas of bosonic particles having an effective mass which is many orders smaller than mass of an electron in vacuum. This allows to study relatively high-temperature phase transitions in low-dimensional bosonic systems. Recently the evidence of K-T (Kosterlitz-Thouless) phase transitions, super fluid behavior of exciton-polaritons in such systems have been reported by many labs[15,16]. However, in the current semiconductor structures the thermalization time is in the picoseconds domain and comparable with the particle life time. Here, one deals with non-equilibrium condensates[17] or which dissipative and optical pumping effects play a crucial role[18]. The characteristic temperature of these condensates are few ten Kelvin which is far above the atomic condensates, but still far below the room temperatures. Recently room-temperature Bose-Einstein condensation of photons have been obtained[19,20]. Here photons are confined in a 2D curved-mirror optical micro resonator filled with dye solution. Thermalization of photon gas is established by thermal contact between the photon and the dye solution using repeated absorption and re-emission processes in the dye solution[21]. Frequent collisions on the time scale much faster than the excited state life time lead to a decoupling of photons and the dye molecules, thus the relevant particles are not polaritons but photons in these experiments.

One has focused a different approach to reach a high (room and above) temperature phase transitions with mixed matter-field states-polaritons in an atomic medium. These dressed state polaritons, where light field is coherently and strongly coupled to a two-level atom. This leads to a bosonic quasi particle which are attractive candidates due to their potentially longer life times[22,23]. In this system thermal equilibrium of coupled atom-light (dressed) states can be achieved experimentally within a nanosecond domain and is limited due to natural life time of two level atoms. Here the optical collisions with buffer gas atoms can lead to thermalization.

The obtained time of thermalization is about ten times shorter than the natural life time at full optical power[24,25]. To overcome this problem, special metallic waveguide with various configurations with the length-up to few mm have to be considered for trapping the polaritons inside. This is similar to waveguides and closed resonators examined for the confinement of microwave radiations[26]. The lifetime of photon-like polaritons trapped in the waveguide can be longer than the thermalization time and is mainly determined by the cavity Q-factor.

In this paper, we have evaluated the energy of trapped polaritons as a function of coordinate z using two potential (1) exact trapping potential U(z) (2) Power law trapping potential $U_{pol}(z)^{\nu}$. We have also evaluated density of state ρ (ε) as a function of energy ε/ħΔ. In another calculation, we have evaluated critical temperature T_C as a function of LB polariton number N_{POL} for three values of trapping power parameter ν. Our evaluated results are in good agreement with other theoretical workers[27,28].

MATERIALS AND METHODS

The thermodynamics of atomic polaritons in a small volume cavity can be obtained by Holstein-Primakoff transformation for atomic excitations[29]. The total Hamiltonian of the atom and quantized field is given by

$$H=H_{rad}+ H_{atom}+ H_{int} \qquad (1)$$

Where H_{rad} is the non-interacting photons in the waveguide. H_{atom} is the Hamiltonian of atomic ensemble and H_{int} is responsible for the interaction of N_{at} two levels atoms with quantized optical fields in the cavity.

$$H = \hbar\sum_{k}[\omega_{ph} f_k^{+} f_k + \omega_{at}\phi_k^{+}\phi_k + k(f_k^{+}\phi_k + \phi_k^{+} f_k)]$$
$$-\frac{\hbar k}{2N_{at}}\sum_{k,k',q}[f_{K+q}^{+}\phi_{k'-q}^{+}\phi_k\phi_{k'} + \phi_k^{+}\phi_{k'}^{+}\phi_{k'-q} f_{k+q}] \qquad (2)$$

Where $f_k(f_k^{+})$ is the annihilation (creation) for the photons absorbed (or emitted), ϕ_k (ϕ_k^{+}) is the annihilation (creation) operators that characterizes excitations (polaritons) of two level atomic ensemble and obeys usual commutations relations for the Bose system

$$k = [\frac{|\wp_{ab}|^2 \omega_L N_{at}}{2\hbar\varepsilon_0 V_M}]^{\frac{1}{2}} \qquad (3)$$

Is the collective atom-field interaction strength, $\wp_{ab}$ is the atomic dipole matrix element, V_M is an effective volume of mode occupation within the region of atom-field interaction. V_M is given by

$$V_M = \int_{cavity} \frac{|\phi(r)|^2}{\max(|\phi(r)|^2)} d^3 r \qquad (4)$$

$$\omega_{at} = \omega_{ph} = \omega_0 + \frac{\hbar^2 k_z^2}{2m_{at}} \qquad (5)$$

ω_0 is the atomic transition frequency. If the number of photons is smaller than the number of atoms then the dispersion relation for polariton state is not modified compared to the uncoupled case then one can diagonalize the total Hamiltonian (2) by using the unitary transformation

$$\Xi_{1,k} = x_k f_k + c_k \phi_k \qquad 6(a)$$

$$\Xi_{2,k} = x_k \phi_k - c_k f_k \qquad 6(b)$$

Where the introduced annihilation operators $\Xi_{1,k}, \Xi_{2,k}$ characterize polariton in the atomic medium, corresponding to two types of elementary excitations which in low density satisfies usual boson commutation relations[30]. Parameters X_K and C_K are real Hopfield coefficients satisfying the condition $X_K^2+C_K^2=1$. This determines the contribution to the photons (C_K) and atomic excitations (X_K) fraction to polarition annihilation operators (6) according to

$$X_k = \frac{1}{\sqrt{2}}\left[1 + \frac{\delta_k}{\sqrt{4k^2 + \delta_k^2}}\right]^{\frac{1}{2}} \qquad 7(a)$$

$$C_k = \frac{1}{\sqrt{2}}\left[1 - \frac{\delta_k}{\sqrt{4k^2 + \delta_k^2}}\right]^{\frac{1}{2}} \qquad 7(b)$$

Taking equations (6), (7) into (2), one can obtain LB Hamiltonian H_{LB} in the form

$$H_{LB} = \hbar \sum_k \Omega_k \Xi^{+}_{2,k} \Xi_{2,k} + \sum_{k,k',q} U_{k,k',q} \Xi^{+}_{2,k+q} \Xi^{+}_{2,k'-q} \Xi_{2,k} \Xi_{2,k'} \qquad (8)$$

Where $\Omega_k = \frac{1}{2}[\omega_{at} + \omega_{ph} - \Omega_R]$. This determines the dispersion relation for LB polaritons. $\Omega_R = (\delta^2 + 4\kappa^2)^{\frac{1}{2}}$ is the Rabi splitting frequency that determines the gap between upper and lower states.

$$U_{k,k',q} = \frac{\hbar\kappa}{2N_{at}}[C_{|k+q|}X_{k'} + C_{k'}X_{|k+q|}]X_{|k'-q|}X_{k'} \qquad (9)$$

This determines the two-body polariton-polariton splitting processes. The effective mass of LB polariton is given by

$$m_{pol} = [\hbar[\frac{\partial^2\Omega_k}{\partial k^2_z}]_{z=0}]^{-1}$$

$$= \frac{2m_{at}m_{ph}\Omega_{Rz}(z)}{(m_{at}+m_{ph})\Omega_{Rz}(z) - (m_{at}-m_{ph})(\Delta + V_{ph}(z)/\hbar)} \qquad (10)$$

Where

$$\Omega_{Rz}(z) = [(\Delta + \frac{V_{ph}(z)}{\hbar})^2 + 4\kappa^2]^{\frac{1}{2}} \qquad (11)$$

This is z-dependence Rabi splitting frequency.

Now Hamiltonian for the LB polariton for small momenta $\frac{\hbar^2 k^2_z}{2m_{pol}} \ll \Omega_{Ro}$

$$H_{LB} \approx \sum_k [\frac{\hbar^2 k^2_z}{2m_{pol}} + U(z)]\Xi_k^{+}\Xi_k \qquad (12)$$

Where U(z) is an effective trapping potential for polaritons defined as

$$U(z) = \frac{1}{2}[V_{ph}(z) - \{(\hbar\Delta + V_{ph}(z))^2 + 4\hbar^2\kappa^2\}^{\frac{1}{2}} + \hbar\Omega_{Ro}] \qquad (13)$$

The last term in brackets of equation (10) specifies the minimal level of potential energy U(z) which is equal to zero, $U(z)_{Z=0}$ at the centre of the trap. From equation (10) under the condition

$\Delta E_n \ll K_\beta T \ll \hbar\Omega_{Ro}$, for photon like polaritons, one can obtain a simple expression for power law potential

$$U(z) \cong U_{pol}\left|z\right|^{\nu} \qquad (14)$$

Where $$U_{pol} = m_{ph}c^2\alpha(\Omega_{Ro} + |\Delta|)/2\Omega_{Ro} \qquad (15)$$

Let us examine the statistical properties of LB polaritons trapped in waveguide cavity. Now, one can treat the polaritons as one dimensional ideal bosones confined in a potential U(z). In the quasi classical approximation, the density of states approaches to

$$\rho(\varepsilon) = \frac{\sqrt{2}}{\pi\hbar}\int_0^{l(\varepsilon)} [\frac{m_{pol}}{\varepsilon - U(z)}]^{\frac{1}{2}} dz \qquad (16)$$

Where $l(\varepsilon) = [\frac{\varepsilon}{U_{pol}}]^{\frac{1}{2}}$ is a characteristic length of localization for LB polaritons with energy ε. For LB-poaritons weakly confined inside the region Z< $Z_{1/2}$, one can use power law approximation (14) of the trapping potential U(z). The Total number of LB polaritons N_{POL} is given by

$$N_{POL} = N_0 + \int \frac{\rho(\varepsilon)d\varepsilon}{\exp[\frac{(\varepsilon-\mu)}{K_\beta T}] - 1} \qquad (17)$$

Where N_0 is the number of ground state polaritons, μ is chemical potential. One finds the critical temperature T_C for which the ground state occupation becomes macroscopically by solving equation (17) at μ =0. For the onset of Bose-Einstein condensation, one finds[3]

$$K_\beta T_C = [\frac{\pi\hbar N_{pol}\nu U^{\frac{1}{\nu}}{}_{pol}}{\sqrt{2m_{pol}}F(\nu)\Gamma(x)\xi(x)}]^{2\nu/(2+\nu)} \qquad (18)$$

Where $$F(\nu) = \int_0^1 \frac{t^{\frac{1}{\nu}} - 1}{\sqrt{1-t}} dt \qquad (19)$$

Ѓ(x) and ξ(x) are Gamma function and Riemann Zeta function for x=1/ν and x=+1/2 respectively. Below the critical temperature the occupation of the ground state is then determined by

$$N_0 = N_{pol}[1-(\frac{T}{T_C})^{\frac{1}{\nu}+\frac{1}{2}}] \quad (20)$$

In the experiments, the average number of LB polaritons is given by

$$N_{pol} = \sum_k \langle \Xi_k{}^{+} \Xi_k \rangle = N_{ph} \quad (21)$$

N_{ph} can be estimated using the phonon-like character of polaritons in the perturbation limit.

RESULTS AND DISCUSSION:

In this paper using the theoretical formalism of I Yu Chestnov etal.[31], we have theoretically studied BEC of trapped atomic polaritons in a biconical waveguide cavity. We have evaluated three important parameters in this paper (a) energy of trapped polaritons as a function of coordinate z for trapping parameter ν =1(b) dependence of density of states ρ(ε) on normalized polaritons energy ε/ЋΔ using two potentials U(z) and U_{Pol} $(z)^{\nu}$ (c) Critical temperature T_C of BEC of atomic polaritons as a function of number of polaritons N_{POL}. In this calculation rubidium atomic densities have been taken. Average number of photons N_{ph} is smaller than average number of atoms N_{at}. $(n_{at})=N_{at}/V_M=10^{16}$ cm^{-3}, occupation volume V_M of the lowest photonic mode (TM_{01} mode)=0.5μm^3. N_{at}=5000 atoms in the biconical waveguide cavity (BWC). Thermal de Broglie wavelength $\Lambda_T = [\frac{2\pi\hbar^2}{m_{pol}K_\beta T}]^{\frac{1}{2}}$ at the temperature of the atomic gas T=530K is macroscopically large i.e $\Lambda = 1.89\mu m$. This is comparable with the magnitude of characteristic length d1 of photonic field localization. In Table T1, we have presented the evaluated results of energy of polariton trapped as a function of coordinate z taking ν=1. The evaluation is done using two potential U(z)/2πЋ and $U_{pol}(z)^{\nu}$/ 2πЋ as a function of z. In both cases, U(z)/ 2πЋ decreases from -70 μm to 0 and increases from 0 to 70 μm. The decrease and increase is faster in the case of second potential. In table T2, we have shown the evaluated results of dependence of density ρ(ε) on normalized polaritons energy ε/ЋΔ using two potentials U(z) and U_{Pol} $(z)^{\nu}$. In both cases, ρ(ε) increases with energy ε/ЋΔ. It was noticed that the increase is faster in the case of power law potential. In table T3, we have presented the evaluated results of critical potential T_C of BEC of trapped atomic polaritons as a function of LB polaritons number N_{POL} using equation (18) for three values of trapping power parameter ν. Since the function ξ(x) shown in equation (18) diverges at x=2, the critical temperature T_C vanishes for increasing trapping power parameter v. T_C is large for v=0.8 and small for v=1.5. We also observed that T_C increases with number of LB polaritons N_{POL}. Our evaluated theoretical results are in good agreement with other theoretical workers[32-34].

CONCLUSION:

From the above theoretical investigations and analysis, we have come across the following conclusions

(1)We have analyzed the problem of BEC of trapped atomic polaritons in a biconical waveguide cavity under the quasi classical condition.
(2)The atomic polaritons formed in the waveguide cavity behaves like 1D ideal gas of bosonic quasi particle.
(3)The polaritons are due to thermalization of coupled atom-light states in the presence of strong field coupling regimes.
(4)We observed high critical temperature of trapped BEC of trapped atomic polaritons in biconical waveguide cavity. The main reason for the high temperature (T_C>530K) is due to the photon like character of the polaritons because of its low effective mass.

TableT1

An evaluated result of energy of polaritons trapped as a function of coordinate z for trapping power parameter v for two potential U(z) and $U_{Pol}(z)^v$

Z(µm)	<-----U(z)/ 2πħ-------------→(THz)	
	U(z)/2πħ(THz)	$U_{pol}(z)^v$/ 2πħ(THz)
-70	12.627	15.346
-60	11.409	13.086
-50	10.584	11.589
-40	9.239	9.654
-30	7.182	8.081
-20	5.486	6.328
-10	2.086	3.106
0	0.054	1.064
10	1.954	2.659
20	4.896	5.122
30	6.995	7.054
40	10.054	9.143
50	11.453	11.896
60	12.086	13.447

70	13.159	15.908

TableT2

An evaluated result of dependence of density of states ρ(ε) on normalized polaritons energy ε/ħΔ. Evaluation has been performed by taking two potential into account U(z) and $U_{Pol}(z)^{v}$

ε/ħΔ	**<-------- ρ(ε) (J^{-1})-------→**	
	U(z) [using eqⁿ(13)]	U_{Pol}(z)[using eqⁿ(14)]
0.0	1.56×10^{21}	1.98×10^{21}
0.1	2.24×10^{21}	3.50×10^{21}
0.2	4.46×10^{21}	4.95×10^{21}
0.3	5.12×10^{21}	5.89×10^{21}
0.4	6.25×10^{21}	6.70×10^{21}
0.5	6.58×10^{21}	6.98×10^{21}
0.6	7.05×10^{21}	7.32×10^{21}
0.7	7.89×10^{21}	7.96×10^{21}
0.8	8.16×10^{21}	8.22×10^{21}
0.9	8.59×10^{21}	8.89×10^{21}
1.0	9.27×10^{21}	9.58×10^{21}
1.5	1.08×10^{22}	1.18×10^{22}

TableT3

An evaluated result of critical Temperature T_C (K) as a function of number of LB polaritons N_{POL}. Results are obtained for three different values of ν =0.8, 1.0 and 1.5

N_{POL}	**<-------------T_C(K)----------------→**		
	ν =0.8	ν =1.0	ν =1.5
100	502.62	427.38	56.98
200	812.85	510.86	82.29
300	1127.56	750.58	116.58
400	1572.58	842.95	186.23
500	1862.08	940.54	230.48
600	2010.35	1260.32	295.20
700	2232.46	1372.59	372.89
800	2492.30	1430.46	427.51
900	2610.32	1510.26	517.80
950	2782.16	1623.10	629.10
1000	2910.85	1700.58	647.23

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