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AN EVALUATION OF DISPERSION RELATION OF PROPAGATION OF ELECTROMAGNETIC WAVE WITH TM MODES IN CARBON NANOTUBE WITH AND WITHOUT ELECTRON ENERGY-BAND EFFECTS

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Abstract: - Using the theoretical formalism of **Li Wei et al (Phys. LettA 333 (2004)) and Afshin Moradi (J. Electromagnetic Analysis & Applications (2010),** we have studied the dispersion relation of TM-mode with and without electron energy band effects. Without including electron energy band effects, the dispersion relation can be studied with linearized hydrodynamic theory with Maxwell equations. This indicates that the TM-mode is very different from TE-mode. Here, the dispersion relation does not approach to well-known dispersion relation of 2D electron-gas. Besides, it also indicates that internal interaction forces play an important role on the dispersion relation of TM-mode. The dispersion relation of TM-mode including electron energy band effects can be studied by means of the semi classical kinectic theory of the electron dynamics. The effect of energy band structure is taken into account for surface plasmon oscillations in the zig-zag and armchair nanotube of metallic character. Our theoretical results also indicate that plasmon waves are not sensitive to the types of metallic nanotube with same radius. Our theoretically evaluated results are in good agreement with those of the other theoretical workers.

Keywords: Single wall carbon nanotubes, lineaized hydrodynamic theory, Nano waveguide, hydrodynamic equations, dispersion relation for surface waves, Transverse electric mode (TE-mode), Transverse magnetic mode (TM-mode), Low frequency electromagnetic wave, Electrostatic collective excitations, Single electron excitation effects, surface plasmon wave, internal interaction forces, Zig-zag and armchair nanotubes.

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1. INTRODUCTION

In an earlier paper [1], we have studied the propagation of electromagnetic wave propagation in carbon nanotubes and presented a method of evaluation of dispersion relation of TE-mode. In this paper, we have presented a method of evaluation of TM-mode. The evaluation has been performed by theoretical formalism of Li Wei etal.[2]. In this formalism, they have moduled the nanotube surface as infinitesimally thin layer of free-electron gas which is described by means of linearzied hydrodynamical theory. General expression of dispersion relation are obtained for the low frequency electromagnetic wave with transverse magnetic mode. This has been achieved by solving Maxweell and hydrodynamic equations with appropriate boundary conditions. Numerical methods were performed and it was observed that TM mode is quite different from TE-mode. These two modes are different for large wave numbers and also for large nanotube radius. We first obtained the dispersion relation for TM mode using hydrodynamical model which does not consider the electron energy- band effects. These effects are valid only for describing dispersion relation for chiral carbon nanotube.

We have studied the energy band effects on the dispersion relation of the surface plasmon wave in single walled carbon nanotube (SWCNTs) of metallic behavior by the classical kinetic theory of the electron dynamics. Earlier[3,4], one focussed on plasmon wave oscillations in a cylindrical electron gas as a simple model of metallic tube. In this case, we have taken the more exact analysis of geometrical effect including the radius and the chiral angle of the nanotube.

From the discovery by Iijim[5] of carbon nanotubes (CNTs), there has been a growing interest in electromagnetic wave propagation in single-wall carbon nanotubes. Since CN can be metals or semiconductors depending upon its radius and the geometrical angle, some important information about the structural and electronic properties can be obtained using electromagnetic probe techniques[6,7] or electron probe techniques[8]. In particular, being very long, a CN can be regarded as a nano waveguide to guide electromagnetic waves. During the past years, different theoretical models have been used to describe the physical properties of CNs. With a classical hydrodynamic model, Yanouleas[9] and Jiang[10] studied the collective excitation behavior of σ and π electrons in single or multi-wall CNs. The collective excitation properties of CNs are quite different from those of well-known graphite sheets. The difference is that the collective excitations in CNs have traditional one-dimensional (1D) characters for small wave number, while exhibiting two-dimensional (2D) behavior for large wave numbers. The quantum dielectric-response theory, taking into account the electron energy band structures in CNs have used[11-13] to describe low frequency electronic excitations in CNs. Beyond the electrostatic excitations, Slepyan etal.[14-16] studied electromagnetic processes in CNs. With

the classical electrodynamics and a semi-classical kinetic theory, they derived the dispersion relation of surface wave in CN's. They found out that the CNs can be used as a waveguide for controlling electromagnetic wave propagation in specified frequency ranges (examples are infrared and optical). They also presented a general quantum mechanical theory of the conductivity of a single wall carbon nanotubes with interband transitions.

Now days, there is much interest to electromagnetic high-frequency properties of carbon nanotubes. This is because of their potential applications in nanoelectronics[17], nanoantennas[18-21], polarizers[22], free electron lasers[23], devices for THz sensing and imaging[24]. As we know that carbon nanotubes (CNs) possess metallic properties therefore special interest has been paid due to their high conductivity at THz frequencies and compared to metal nanowires[25]. By this reason, their applications seem to be promising in THz and infrared ranges due to noticeable lower losses compared to other conductive materials.

One of the most important electromagnetic property of metallic CNTs is a capability to support propagation of strongly delayed surface waves[26,27]. It is caused by a very high kinetic inductance of thin single –wall CNTs[28]. It makes electromagnetic (EM) wave propagation in CNTs strongly different compared to transmission lines, made of usual bulk metals. For description of electromagnetic properties of metallic CNTs, very often the model of impedance cylinder and effective boundary condition is used[29]. The model of impedance cylinder takes into account quantum properties of CNTs via the complex surface frequency-dependent conductivity. This model was applied for theoretical study of CNT transmission lines and interconnects. These structures are composed of closely packed bundles of parallel identical metallic CNTs. It was applied for studying two-dimensional periodic arrays of single wall metallic CNTs.

2. MATERIALS AND METHODS:

One models a single wall carbon nanotube as an infinitesimally thin and infinitely long cylindrical shell with a radius a. One assumes that the valence electrons can be considered as free electron gas distributed uniformly over the cylindrical surface. Let the density per unit area be n_0. One uses cylindrical coordinate $\mathbf{r}=(\rho,\phi,z)$. Consider an electromagnetic wave with frequency ω propagating along the nanotube z-axis. The homogeneous electron gas will be perturbed by the electromagnetic wave and can be regarded as a charged fluid with velocity field $\mathbf{u}(\mathbf{r}_S,t)$ and the perturbed density (per unit area) $n_1(\mathbf{r}_S,t)$. $\mathbf{r}_S=(\phi,z)$ is the coordinate of a point at the cylindrical surface of the nanotube. Velocity field $\mathbf{u}$ has only tangential components to the nanotube surface. Based on the linearized hydrodynamic model[30], the electronic excitations on the cylindrical surface can be described to the continuity equation

$$\frac{\partial n_1(r^{\rightarrow}_S,t)}{\partial t}+n_0\nabla_{\square}.u^{\rightarrow}(r_S^{\rightarrow},t)=0 \qquad (1)$$

And the momentum –balance equation is given by

$$\frac{\partial u^{\rightarrow}(r_s^{\rightarrow},t)}{\partial t}=-\frac{e}{m_e}E_{\square}^{\rightarrow}(r_S^{\rightarrow},t)-\frac{\alpha}{n_0}\nabla_{\square}n_1(r_S^{\rightarrow},t)+\frac{\beta}{n_0}\nabla_{\square}(\nabla^2{}_{\square}n_1(r_S^{\rightarrow},t)) \qquad (2)$$

Where $E^{\rightarrow}_{\square}=E_z e^{\rightarrow}_z+E_\phi e^{\rightarrow}_\phi$ is the tangential component of the electromagnetic field. n_1 is the charge density polarization of the electron gas, e is the electronic charge and m_e is the electron mass. Here $\nabla_{\square}=e^{\rightarrow}_z\frac{\partial}{\partial z}+e^{\rightarrow}_\phi\alpha^{-1}\frac{\partial}{\partial\phi}$ only differentiates tangentially to the nanotube surface. The first term on the right hand side of equation (2) is the force on electrons on the nanotube due to the tangential component of the electric field the second and third terms may be regarded as parts of the internal interaction force in the electron gas. Here $\alpha=(V_F^2/2)$ is the speed of propagation of density disturbances in the electron gas with $V_F=(2\pi n_0 a^2{}_\beta)^{\frac{1}{2}}v_\beta$ being the Fermi velocity of the 2D electron gas and $\beta=(a_\beta v_\beta)^2/4$ describes single electron excitations in the electron gas. Here a_β and v_β are the Bohr radius and Bohr velocity respectively. We have neglected the second and third terms in the calculation as it was neglected in the works of Yannouleas etal[5].

The electric field vector **E(r,**t) and the magnetic field vector **B(r,**t) can be expanded in the following Fourier forms

$$E(r,\phi,z,t)=\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}dqE^{\rightarrow}_m(r,q)e^{i(m\phi+qz-\omega t)} \qquad (3)$$

$$B(r,\phi,z,t)=\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}dqB^{\rightarrow}_m(r,q)e^{i(m\phi+qz-\omega t)} \qquad (4)$$

Using Maxwell's equations, one can obtain the following Helmholtz equations for the z-components E_{zm} and B_{zm} of the expanding coefficients $\mathbf{E}_{zm}$ and $\mathbf{B}_{zm}$

$$\frac{d^2E_{zm}}{dr^2}+\frac{1}{r}\frac{dE_{zm}}{dr}-(\kappa^2+\frac{m^2}{r^2})E_{zm}=0 \qquad (5)$$

And $$\frac{d^2B_{zm}}{dr^2}+\frac{1}{r}\frac{dB_{zm}}{dr}-(\kappa^2+\frac{m^2}{r^2})B_{zm}=0 \qquad (6)$$

Where $\kappa^2=q^2-k^2$, $k=\frac{\omega}{c}$ 7(a)

k is wave number and c is velocity of light. We have assumed that the propagation of electromagnetic waves are in the infrared regime so that $k \ll q$.

By eliminating the velocity field **u**(r,t), one can obtain the following equations from equation(1) and (2)

$$\frac{\partial^2 n_1(r_s^{\rightarrow},t)}{\partial t^2}=\frac{en_0}{m_e}\nabla_{\perp}.E_{\perp}^{\rightarrow}(r_s^{\rightarrow},t)+\alpha\nabla_{\perp}^2 n_1(r_s^{\rightarrow},t)-\beta\nabla_{\perp}^2(\nabla_{\perp}^2 n_1(r_s^{\rightarrow},t)) \qquad (8)$$

Upon solving equation (8) by means of space-time Fourier transforms for the induced density $\mathbf{n_1(r}_S,t)$ on the cylindrical surface, one finds

$$n_1(\phi,z,t)=\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}dqN_m^{\rightarrow}(q)e^{i(m\phi+qz-\omega t)} \qquad 9(a)$$

Where $$N_m=-i\frac{en_0}{m_e}\frac{1}{W_m}q_m^{\rightarrow}.E_{\perp m}^{\rightarrow} \qquad 9(b)$$

With $$q_m^{\rightarrow}=qe_z^{\rightarrow}+(\frac{m}{a})e_{\phi}^{\rightarrow} \qquad 9(c)$$

$$W_m=\omega^2-\alpha q_m^{\ 2}-\beta q_m^{\ 4} \qquad 9(d)$$

To solve equations (5) and (6), one has to provide appropriate boundary conditions. With the induced density, these boundary conditions can be written as

$$E_{rm}(a)_{r>a}-E_{rm}(a)_{r<a}=-\frac{en_0}{\varepsilon_0} \qquad 10(a)$$

$$(E_{\square m}^{\rightarrow})_{r>a} - (E_{\square m}^{\rightarrow})_{r<a} = 0 \qquad 10(b)$$

And $$B_{rm}(a)_{r>a} - B_{rm}(a)_{r<a} = 0 \qquad 10(c)$$

where ε_0 is the permittivity of free space. Equation 10(a) indicates that due to the polarization of the electron gas on the nanotube surface, the radial component of the electric field is discontinuous at the cylinder at r=a.

Dispersion relation of TM-mode without including electron energy-band effects

For the TM mode, the longitudinal magnetic field is zero, ie. B_Z=0. From equation (5), the longitudinal electric field can be expressed by

$$E_{zm}(r) = C_m I_m(\kappa r), (r < a) \qquad (11)$$

And $$E_{zm}(r) = D_m K_m(\kappa r), (r > a) \qquad (12)$$

Similarly, with Maxwell equations, we find the expressions of E_{rm}, $E_{\phi m}$ and $E_{\phi m}$ as follows

$$E_{rm}(r) = -i\frac{q}{\kappa^2}\frac{dE_{zm}(r)}{dr} \qquad (13)$$

$$E_{\phi m}(r) = \frac{mq}{\kappa^2 r}E_{zm}(r) \qquad (14)$$

And $$B_{rm}(r) = (1-\frac{q^2}{\kappa^2})(\frac{m}{\omega r})E_{zm}(r) \qquad (15)$$

In this case, the Fourier Coefficent N_m of the induced density is written as

$$N_m = -i\frac{en_0}{m_e}\frac{1}{W_m}(qE_{zm} + \frac{m}{a}E_{\phi m}) \qquad (16)$$

Combining the boundary conditions (Equations 10(a), 10(b) & 10(c)) with equations (11)-(16), the dispersion relation of TM mode can be written as

$$\omega^2 - \alpha(\kappa^2 + \frac{\omega^2}{c^2} + \frac{m^2}{a^2}) - \beta(\kappa^2 + \frac{\omega^2}{c^2} + \frac{m^2}{a^2})^2$$

$$= \frac{\Omega^2{}_p}{(\kappa a)^2}(m^2 + \kappa^2 a^2) I_m(\kappa a) K_m(\kappa a) \qquad (17)$$

With the dimensionless variable $y = \frac{\omega}{\Omega_p}$ and $x = \kappa a$, the dispersion relation in the low frequency case can be reduced to

$$y^2 = \alpha_1(x^2 + m^2) + \beta_1(x^2 + m^2)^2 + \frac{1}{x^2}(m^2 + x^2) I_m(x) K_m(x) \qquad (18)$$

The results are shown in table T1, T2 and T3 in this paper.

Dispersion relation of TM-mode including electron energy-band effects

Here, one considers both zig-zag (m, 0) and armchair (m, m) nanotubes as infinitesimally thin and infinitely long cylindrical shells of radius r_c with its axis along the z-direction. CNT consists of π-electrons superimposed with equlibrium densities (per unit area) n_0. One assumes that in equlibrium the π-electron fluid has no velocity and n is the perturbed density (per unit area) of fluid, produced by the π-electrons themselves under the action of the electric field generated by the fixed positive ions of the lattice. Here, one considers the surface plasmon waves with TM modes. The tube radius of the CNT is given by

$$r_c = \frac{a_0}{2\pi}\sqrt{m^2 + mn + n^2} \qquad (19)$$

Where m and n are integers, $a_0 = \sqrt{3} b_0$ is the lattice constant of the graphite sheet and $b_0 = 1.42 A^0$ is the distance between the nearest-neighboring carbon atoms. A SWCNTs is metallic if $|m - n| = 3q$, where q=0,1,2,3----. The armchair nanotubes are always metallic, whereas zig-zag nanotubes are metallic only if m=3q with q=1,2---

The dispersion equation for TM mode with electron energy-band effects can be written as

$$(\omega + i\gamma)\omega = \alpha(\kappa^2 + \frac{m^2}{r^2_c}) + \omega^2{}_p r_c{}^2 (\kappa^2 + \frac{m^2}{r^2_c}) I_m(\kappa r_c) K_m(\kappa r_c) \qquad (20)$$

Where $I_m(x)$ and $K_m(x)$ are modified Bessel's functions, $\alpha = V_F^2/2$ is the speed of propagation of density disturbances in a uniform 2D homogeneous electron fluid. γ is friction coefficent.

$\kappa^2 = q^2 - \frac{\omega^2}{c^2}$, c is light speed. The parameter κ is the real quantity in the region $\frac{\omega}{q} < c$. $\omega_p = [\frac{e^2 n_0}{\varepsilon_0 r_c m_{eff}}]^{\frac{1}{2}}$ is the eigen-frequency of the π-electron gas layer in metallic SWCNTs. The solution of equation (20) yield complex frequencies $\omega = \omega_r + i\omega_i$. It may be observed that the imaginary part will be $\omega_i = -\frac{\gamma}{2}$. Now by writing $\omega = \omega_r - i\frac{\gamma}{2}$, the solution for finite damping will be of the form

$$\omega = [\alpha(\kappa^2 + \frac{m^2}{r_c^2}) + \omega^2{}_p r^2{}_c (\kappa^2 + \frac{m^2}{r_c^2}) I_m(\kappa r_c) K_m(\kappa r_c) - \frac{\gamma^2}{4}]^{\frac{1}{2}} - i\frac{\gamma}{2} \qquad (21)$$

The friction coefficent[31] is the inverse of the relaxation time $\tau=3x10^{-12}$s. The dispersion characteristics of the surface waves in the system depend upon nanotube geometry. This includes the radius and chiral angle of the nanotube. It also depends on the wave number, angular momentum and the friction coefficent. It is observed that by increasing friction coefficent, the dispersion curves shift to lower frequencies, then one set γ=0.

At this stage, from equation (21), one can see for investigation the dispersion characteristics of the zig-zag and armchair SWCNTs. One has to give the values of n_0/m_{eff}. The parameter n_0/m_{eff} takes into account the influence of the atomic crystal field. By using the semiclassical model of the π-electron dynamics, one obtains the following estimation

$$\frac{n_0}{m_{eff}} = \frac{2v_F}{\pi^2 \hbar r_c} \qquad (22)$$

Where v_F is the velocity of the electrons at the Fermi level

$$v_F = \frac{3\gamma_0 b_0}{2\hbar} \qquad (23)$$

Here γ_0 is the characteristic energy of the graphene lattice (γ_0 =2.7-3eV), b_0 is the distance3 between the nearst-neighbouring carbon atoms $b_0=1.42A^0$. The value of v_F comes out = (0.9-1) $x10^6$ m/s. The equation (23) holds for zig-zag nanotubes with m=3q<60, for armchair nanotubes with m<50 and for chairl nanotubes with 2n+m=3q. In the range of validity of equation (23) the parameter decreases as the nanotube radius increases.

To see clearly the nergy band effects on the dispersion relation of the surface waves in SWCNTs, one takes the long and short wavelength limits of the equation (21). For $\kappa r_c \to \infty$, using the well known asymptotic expressions[32], we have

$$I_m(x) = \frac{e^x}{\sqrt{2\pi x}} \qquad 24(a)$$

$$K_m(x) = \sqrt{\frac{\pi}{2x}} e^{-x} \qquad 24(b) \qquad \text{(with finite m)}$$

The dispersion relation can be written approximately as

$$\omega^2 = \alpha\kappa^2 + \frac{e^2 v_F}{\varepsilon_0 \pi^2 \hbar} \frac{\kappa}{r_c} \qquad (25)$$

This shows that earliar the dispersion relation is independent of the geometrical effects of the tube, the right hand side of equation (25) the dispersion relation depends strongly on the radius of the tube. It appears as the nanotube radius r_c increases the value of ω decreases. In the opposite limit $\kappa r_c \to 0$, where the phase velocity of the surface plasmon is comparable to the velocity of light, surface plasmon oscillations couple with the electromagnetic wave and retardation effects are present. Retardation effects in low-dimensional plasmons were investigated[33]. If one neglects the retardation effects by using well known expressions of Bessel functions

$$I_m(x) = \frac{1}{\Gamma(m+1)} (\frac{x}{2})^m \qquad 26(a)$$

$$K_m(x) = \frac{\Gamma(m)}{2} (\frac{2}{x})^m \qquad (\text{for } m \neq 0) \qquad 26(b)$$

And $$K_0(x) = \frac{\ln 1.123}{x} \qquad \text{(for m=0)} \qquad 26\text{ (c)}$$

Then, one obtains for m=0

$$\omega[m=0, \kappa=0] = [\frac{4e^2 v_F}{\varepsilon_0 \pi^2 \hbar} \ln(\frac{1.123}{\kappa r_c})]^{\frac{1}{2}} \kappa \qquad (28)$$

This is a quasiacoustic mode and for m $\neq 0$, one gets

$$\omega^2 = \frac{\alpha}{r_c^2} m^2 + \frac{e^2 v_F}{\varepsilon_0 \pi^2 \hbar r_c^2} m \qquad (29)$$

3. RESULTS AND DISCUSSION:

In this paper, using the theoretical formalism of Li Wei etal.[2] and Afshin Moradi[34], we have evaluated the diepersion relation (ω/Ω_p) for Tm modes as a function of variable κa without including electron energy band effects and also including electron energy band effects respectively. We have evaluated dispersion relation of dimensionless frequency (ω/Ω_p) for TM mode as a function of dimensionless parameter κa for nanotube diameter a=5nm and different values of m. The evaluated results are shown in table T1. From our evaluate results, it appears that the character of dispersion relation of the TM-mode is quite different from that of TE-mode. Here, the values of (ω/Ω_p) decreases as a function of κa for all values of m. At κa=0, its value is large for all values of m starting from m=0 to m=4. Afterwards, it decreases and decreases very sharply. After κa=6 decrease is slower for all values of m. In another calculation, we have evaluated (ω/Ω_p) as a function of κa for different values of nanotube diameter 'a' for fixed values of m=0. The results are shown in table T2. From our evaluated results, we observe that the value of (ω/Ω_p) first decreases and then increases and increases very sharply. The value decreases upto value of κa=5 and then increases. The increase is large for a=2nm and small for a=15nm. It is also noticed that unlike the TE-mode, the dispersion relation of TM- mode does not approach to the well-known dispersion relation fot the 2D electron gas in the limit κa $\rightarrow \infty$. In table T3, we have evaluated (ω/Ω_p) as a function of κa for fixed nanotube radius a=5nm and m=0 for three cases (a) $\alpha_1 \neq 0, \beta_1 \neq 0$ (b) $\alpha_1 = 0, \beta_1 \neq 0$ (c) $\alpha_1 = 0 = \beta_1$ using equation (18).From our evaluated results, it appears that (ω/Ω_p) is large in the case of (a) for all values of κa starting from 0 to 60 and very small in the case of (c) for all values of κa. These results give the influence of the internal interaction forces of the electron gas on the dispersion relation of TM-modes. On comparing this calculation with TE-modes, it can be seen that the internal interaction forces play an important role on the dispersion relation of TM-modes. If these forces are not included, the frequency will decrease rapidly as the wave number increases[35]. The above three calculations were performed without including the electron energy band effects.

We have included electron energy band effects for zig-zag (m, o) and armchair (m, m) nanotubes. These are considered as infinitely thin and infinitely long cylindical shells of radius r_c with its axis along the z-direction.The dispersion relation including electron energy band effects

were calculated using equations (25), (28) and (29). The results are shown in table T4, T5, T6, T7 and T8. In table T4, wehave shown the evaluated results of dispersion curve ω (eV) as a function of q (A^{0-1}) of surface waves for different nanotube geometries for m=0 and γ=0. This gives the electron energy band effects on the dispersion relation of plasmon waves in the system. These results were obtained for zig-zag (12, 0), (27, 0) and armchair (9, 9), (15, 15) nanotubes respectively. Our theoretically evaluated results indicate that ω (eV) increases with q (A^{0-1}) for both varities of nanotubes. The value is large foe zig-zag (12, 0) and small for armchair (15, 15) nanotubes. In table T5, we repeated the above calculation for m=1 and γ=0 and similar results were obtained. The value is large or zig-zag (12, 0) and small for armchair (15, 15) nanotubes. The radius of zig-zag nanotube is r_c=1.056nm and for armchair r_c=1.017nm. This indicates that behavior of plasmon wave is not sensitive to the types of metallic nanotubes with same radius. In table T6, we have shown the results of ω (eV) as a function of q (A^{0-1}) for three values of electron beam velocities by considering the expression ω=vq. We have taken zig-zag nanotube (27, 0) and m=0, γ=0. The three electron beam velocities are $v=4x10^6$m/s, $v=1.5x10^6$m/s and $v=0.93x10^6$m/s. Our theoretical results indicate that ω (eV) as a function of q (A^{0-1}) increases very fast for $v=4x10^6$m/s and slow for $v=0.93x10^6$m/s. The velocity of the electron beam is equal to phase velocity of the surface plasmon modes. The electron beam is in synchronization with the surface wave and they interact with each other and instability occurs between them. This also indicates that surface waves in the system can only be excited by applying some relativistic electron beam which speed is about 10^6 m/s. In table T7, we have shown the evaluated results of E_{zm}/E_{oz} as a function of (r/r_c) for m=0, m=1 and m=2 for vacuum ($r<<r_c$). This gives the surface TM mode of a nanotube as a function of radial coordinate r. Our theoretically evaluated results indicate that (E_{zm}/E_{oz}) increases with (r/r_c). The value is large for m=0 and small for m=2. The value becomes maximum at (r/r_c)=1.0 for each value of m. In table T8, we have repeated the calculation for vacuum($r>>r_c$). Here, (E_{zm}/E_{oz}) decreases with (r/r_c) for each value of m. The value decreases from (r/r_c) =1.0 to 2.0 for each m. Some more works[36-52] in this field also reveals the similar behavior.

4. CONCLUSION:

From the above theoretical investigation and analysis, we come across the following conclusion

- Dispersion relation of TM mode is quite different from TE-mode. TM-mode does not approach to well-known dispersion relation of the 2D-electron gas in the limit $\kappa a \to \infty$.
- The internal interaction forces play an important role on the dispersion relation for the TM-mode. If these forces are not included the frequency will decrease rapidly as the wave number increases.

- If one does not include electron energy band effects then linearzied hydrodynamic model along with Maxwell equations can be used to study the dispersion relation of TM-mode.
- The electron energy band effects can be studied with the help of zig-zag (m, 0) and armchair (m, m) nanotubes. The conduction electron of the system are modelled by an infinitesinally thin layer of free electron gas which is described by means of the semiclassical kinetic theory of the electron dynamics. The propagation of surface plasmon wave in metallic single-walled carbon nanotube is modelled within the framework of classical electrodynamics.
- The results obtained in this paper make one's to believe that the hydrodynamic theory with semiclassical model is appropriate for studies of plasmon oscillations in CNTs for different nanotubes geometries. This also indicates that it can be a good theoretical formalism of carbon nanotube as optical nano waveguides.

Table T1: An evaluated result of the dispersion relation $(\frac{\omega}{\Omega_p})$ of TM-mode for carbon nanotube with radius a=5nm and m=0, 1, 2, 3 and 4 as a function of κa.

κa	$(\frac{\omega}{\Omega_p})$ <------- ---------->				
	m=0	m=1	m=2	m=3	m=4
0.5	4.68	5.20	5.12	4.85	4.75
1.0	4.32	4.58	4.53	4.62	4.32
2.0	3.97	4.27	4.20	4.40	4.00
4.0	3.84	3.90	3.86	4.08	3.67
5.0	3.60	3.76	3.60	3.75	3.43
6.0	3.42	3.68	3.54	3.52	3.29
7.0	3.27	3.38	3.29	3.41	3.06
8.0	2.94	3.10	3.00	3.16	2.73
9.0	2.86	2.98	2.87	2.89	2.54
10.0	2.70	2.84	2.80	2.72	2.49

12.0	2.62	2.67	2.62	2.53	2.35
14.0	2.42	2.50	2.48	2.40	2.27
15.0	2.30	2.42	2.40	2.35	3.20

TableT2: An evaluated result of the dispersion relation $(\frac{\omega}{\Omega_p})$ of TM-mode for carbon nanotube with radius a=2, 5, 10 and 15nm for m= 1 as a function of κa.

κa	$(\frac{\omega}{\Omega_p})$ (m=1)			
	a=2nm	a =5nm	a =10nm	a=15nm
0	1.58	1.42	1.38	1.22
5	0.42	0.82	0.72	0.86
10	0.87	0.76	0.67	0.78
15	1.27	1.08	0.75	0.68
20	1.47	1.32	0.89	0.65
25	1.68	1.46	1.18	0.75
30	1.82	1.58	1.30	0.80
35	2.17	1.62	1.40	0.94
40	2.23	1.68	1.45	1.06
45	2.76	1.78	1.67	1.34
50	2.89	2.08	1.85	1.47
60	3.15	2.34	1.98	1.59

TableT3: An evaluated result of the dispersion relation $(\frac{\omega}{\Omega_p})$ of TM-mode for a=5nm and m=0 for (a) $\alpha_1 = \beta_1 \neq 0$ (b) $\alpha_1 = 0, \beta_1 \neq 0$ (c) $\alpha_1 = \beta_1 = 0$ as a function of κa.

κa	$(\frac{\omega}{\Omega_p})$ (a=5nm and m=0)		
	(a) $\alpha_1 = \beta_1 \neq 0$	(b) $\alpha_1 = 0, \beta_1 \neq 0$	(c) $\alpha_1 = \beta_1 = 0$
0	1,65	1.60	1.48
5	0.56	0.50	0.42
10	0.64	0.42	0.33
15	0.82	0.36	0.27
20	0.97	0.30	0.21
25	1.12	0.24	0.18
30	1.68	0.20	0.15
35	1.84	0.25	0.14
40	2.10	0.29	0.10
45	2.28	0.34	0.08
50	2.40	0.42	0.06
55	2.58	0.56	0.05
60	2.74	0.68	0.04

Table T4: An evaluated results of dispersion relation ω (eV) as a function of wave number q(A^{-0}) for different nanotube geometries for m=0 and γ=0. This shows the electron energ band effects on the dispersion relation of plasmon waves in the system

q(A^{-0})	<------ ω (eV) (m=0 and γ=0)---------→			
	Zig-Zag(12,0)	Zig-Zag(27,0)	Armchair(9,9)	Armchair(15,15)
0.00	0.00	0.00	0.00	0.00
0.05	0.89	0.05	0.67	0.52
0.10	1.52	0.32	1.24	1.14
0.15	1.87	0.57	1.67	1.38
0.20	2.12	0.88	1.85	1.57
0.25	2.56	1.19	2.08	1.84
0.30	2.92	1.58	2.47	2.05
0.35	3.20	1.79	2.75	2.47
0.40	3.58	1.96	2.92	2.78
0.45	3.77	2.27	3.17	2.97
0,50	4.16	2.48	3.52	3.09
0.55	4.38	2.67	3.67	3.22
0.60	4.66	2.86	3.88	3.47
0.65	4.88	3.22	4.02	3.69
0.70	5.16	3.55	4.38	3.89

Table T5: An evaluated results of dispersion relation ω (eV) as a function of wave number q(A^{-0}) for different nanotube geometries for m=1 and γ=0. This shows the electron energ band effects on the dispersion relation of plasmon waves in the system

q(A^{-0})	<------ ω (eV) (m=0 and γ=0)---------→			
	Zig-Zag(12,0)	Zig-Zag(27,0)	Armchair(9,9)	Armchair(15,15)
0.00	2.58	2.05	1.17	1.14
0.05	2.64	2.12	1.25	1.22
0.10	2.73	2.27	1.36	1.32
0.15	2.79	2.34	1.44	1.39
0.20	2.86	2.48	1.52	1.48
0.25	3.08	2.57	1.64	1.61
0.30	3.12	2.68	1.72	1.69
0.35	3.36	2.75	1.76	1.75
0.40	3.58	2.82	1.84	1.80
0.45	3.75	2.89	1.92	1.89
0,50	4.02	2.95	2.08	2.02
0.55	4.28	3.05	2.17	2.14
0.60	4.67	3.12	2.29	2.26
0.65	4.88	3.24	2.35	2.31
0.70	5.05	3.33	2.48	2.42

Table T6: An evaluated results of dispersion curve ω (eV) as a function of wave number q(A^{-0}) for different values of electron beam velocities v keeping m=0 and γ=0 fpr zig-zag nanotube (27,0) writing ω=vq

q(A^{-0})	<--ω (eV) (m=0 and γ=0)---------→ (Zig-zag(27,0))		
	V=4x10^6m/s	V=1.5x10^6m/s	V=0.93x10^6m/s
0.00	0.00	0.00	0.00
0.05	0.89	0.52	0.25
0.10	1.74	0.87	0.57
0.15	2.56	1.24	0.79
0.20	3.29	2.48	1.14
0.25	4.65	2.76	1.39
0.30	5.86	2.89	1.53
0.35	6.49	3.16	1.77
0.40	7.26	3.58	2.11
0.45	8.16	3.97	2.47
0.50	8.87	4.12	2.73
0.55	9.28	4.74	2.99
0.60	9.92	5.32	3.19
0.65	10.16	6.54	3.33
0.70	10.48	7.09	3.85

Table T7: An evaluated results of E_{zm}/E_{0z} as a function of r/r_c for m=0, m=1 and m=2 for vacuum ($r<<r_c$). This gives the surface TM modes of a nanotube as a function of radial coordinate r.

(r/r_c)	E_{zm}/E_{0z} (Vacuum $r<<r_c$)		
	m=0	m=1	m=2
0.00	0.425	0.000	0.000
0.05	0.438	0.012	0.004
0.10	0.446	0.056	0.008
0.15	0.452	0.098	0.016
0.20	0.468	0.124	0.052
0.25	0.470	0.138	0.084
0.30	0.479	0.146	0.123
0.35	0.482	0.159	0.135
0.40	0.488	0.164	0.146
0.45	0.496	0.178	0.154
0.50	0.502	0.182	0.166
0.55	0.509	0.195	0.173
0.60	0.512	0.202	0.182
0.70	0.518	0.224	0.195
0.80	0.522	0.243	0.201
0.90	0.526	0.256	0.223
1.00	0.535	0.284	0.242

Table T8: An evaluated results of E_{zm}/E_{0z} as a function of r/r_c for m=0, m=1 and m=2 for vacuum ($r>>r_c$). This gives the surface TM modes of a nanotube as a function of radial coordinate r.

(r/r_c)	E_{zm}/E_{0z} (Vacuum $r>>r_c$)		
	m=0	m=1	m=2
1.00	0.562	0.354	0.245
1.05	0.540	0.345	0.234
1.10	0.534	0.338	0.227
1.15	0.523	0.330	0.216
1.20	0.514	0.317	0.204
1.25	0.486	0.310	0.195
1.30	0.462	0.292	0.189
1.35	0.458	0.280	0.176
1.40	0.447	0.276	0.166
1.45	0.432	0.269	0.154
1.50	0.426	0.262	0.147
1.55	0.408	0.255	0.136
1.60	0.393	0.242	0.125
1.70	0.378	0.237	0.105
1.80	0.356	0.186	0.0.087
1.90	0.308	0.154	0.052
2.00	0.286	0.148	0.043

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