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ANALYSIS OF PERFORATED PLATES BY FINITE ELEMENT METHOD

PRAMOD V. DESHMUKH

Principal, Dr. Panjab rao Deshmukh Polytechnic, Amravati

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Abstract: Analysis of perforated plate by finite element method is the subject of present paper. Hinton's eight noded isoparametric plate element based on thick plate theory, is utilised for the analysis. The plate theory, in terms of transverse deflection and rotations of the normal about X and Y axes, involves only first derivative in energy expression and hence only C^o continuity is needed in shape functions. The bending, twisting and shear energies are evaluated using uniform 2x2 Gauss integration. The most apparent feature of the isoparametric elements are sides that may be curved and their special coordinate system. They are useful in modelling structures with curved edges and in grading a mesh from course to fine. Hence Hinton's eight noded isoparametric plate element is thought to be most suitable for the analysis of perforated plates. A computer programme is developed to analyse a perforated plate with different boundary conditions.

Keywords: Finite element, Hinton's element, isoparametric, Gauss integration, Mindlin Plate theory, nodes.

Corresponding Author: MR. PRAMOD V. DESHMUKH

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INTRODUCTION

Perforated plates, ie., plate with more than one hole arise in design problems of civil, mechanical and aeronautical engineering. Extensive work have been carried out for plates with single hole but very less attention has been paid to the plates with more than one hole.

With the advent of digital computer, the intractability of the complex continuum problems become easier to anlyse by numerical methods. Willim Bradley (11) in 1956 solved the problem of plate with rectangular opening located anywhere in a plate by finite difference method. V.L. Shah (14) has solved the case of eccentric rectangular opening in a rectangular plate by finite difference method and verified his results by conducting experiments on similar plates. Williams and Chapman (15) in 1969 and Basu and Dawson (16) have derived a finite difference formulation of libore and Bartdorfs equation.

The finite element method is a numerical procedure for solving a continuum mechanics problem with an accuracy acceptable to engineers. M.E1-Hashimy (B) in 1956 solved the case of a finite square plate with circular opening subjected to uniformly distributed load. He has solved the problem by using triangular element with polar coordinates. Zienkiewicz (3) has used triangular element to find the stresses in a square plate with circular opening. Reaz A. Choudhari and Paul Seide (7) have developed a co type triangular element formulation in orthogonal curvilinear coordinates, based on assumptions of transverse inextensibility and transverse load.

It is observed that, no literature is available on the analysis of plate with more than one hole subjected to transverse load. Hence, eight noded isoparametric plate element, based on thick plate theory in which Love Kirchoff's hypothesis is violated, is used to analyse transversely loaded simply supported perforated square plate.

The introduction of isoparametric elements have tremendously enhanced the flexibility and adaptability of finite element method for practical problems. Their most apparent features are sides that may be curved and their special coordinates (?,?) system (Curvilinear co-ordinate system). Thin plate theory requires second derivative in energy expression and it is necessary to have elements of C1 continuity. Hence, isoparametric formulation using thin plate theory is not practical. Mindlin developed his theory for thick plates by including transverse shear deformation. Hinton (17) adopted Mindlin plate theory to develop eight noded isoparametric element in cartesian coordinates. Thus, transverse displacement and rotations of the normal about X and Y axes are assumed nodal d.o.f. The purpose of using Mindlin plate theory is that only first derivative appears in the energy expression and therefore Co continuity of the shape functions is required.

Considering the above aspect, Hinton's element is adopted for the analysis of perforated plate. This element enhanced the applicability of finite element method for modelling perforated plates with various boundary conditions. The element has 3 d.o.f per node, i.e., transverse displacement and rotations of the normal about X and Y axes, resulting in 24 x 24 stiffness matrix. The bending, twisting and shear energies are evaluated by 2 x 2 uniform Gauss integration.

2. PLATE THEORY

The coordinate axes X, Y and Z (Cortesion Co-ordinate System) for a plate are shown in fig. 4.1. The displacement of any point at a distance z from the middle plane are -

$u(z) = z.\,\theta y \quad v(z) = -z.\,\theta x \quad w(z) = w$ ... (1)

Where, θx and θy are the rotations of the normal about X and Y axes respectively, and are treated as positive when clockwise about X and Y axes.

The curvatures,

$\{X\} = \{Xx \quad Xy \quad Xxy\}^T$

and shear strains,

$\{\Upsilon\} = \{\Upsilon xz \quad \Upsilon yz\}^T$

are written as,

$Xx = \partial\theta y / \partial x$

$Xy = \partial\theta x / \partial y$

$Xxy = [\partial\theta y/\partial y - \partial\theta x/\partial x]$

$\Upsilon xz = \theta y + \partial w / \partial x$

$\Upsilon xz = -\theta x + \partial w / \partial y$

The moments, $\{M\} = \{Mx\ My\ Mxy\}^T$

and shear forces, $\{\theta\} = \{\theta x\ \theta y\}^T$

are expressed as -

$$\{M\} = [D1]\ \{X\}$$

$$\{\theta\} = [D2]\ \{\Upsilon\}$$

with [D1] and [D2] as elasticity matrices.

3. FORMULATION

The elements has eight nodes at corners and midsides. It has 24 nodal degrees of freedom, three at each node e.g. displacement w and normal rotations θx and θy. The displacement variation over element domain is described in terms of shape functions and nodal displacement as

$$W = \Sigma\ Ni\ Wi, \quad \theta x = \Sigma\ Ni\ \theta xi$$

$$\theta y = Ni\ \theta yi$$

Shape functions Ni are expressed in local curvilinear coordinates ξ and η.

The element geometry is defined in terms of nodal coordination as

$x = \Sigma\ Ni\ xi,$

$$y = \Sigma\ Ni\ yi,$$

Using various equations mentioned above and derivative transformations, the curvatures and shear strains at a point in element are related to nodal displacement vector $\{\delta e\}_{(24 \times 1)}$ as -

$$\{X\} = [B_1]\ \{\delta e\}$$

$$\{\Upsilon\} = [B_2]\ \{\delta e\}$$

The element strain energy is –

$$U = \frac{1}{2} \, \tilde{\delta}e^{T} \left[\int_A \bar{B}_1^{T} \, \bar{D}_1 \bar{B}_1 \, da + \int_A \bar{B}_2^{T} \, \bar{D}_2 \bar{B}_2 \right] \tilde{\delta}e$$

The integration is carried out using 2x2 Gauss integration scheme. Thus, the element stiffness matrix (24 x 24) is.

$$[Ke] = \sum^{2} \sum^{2} W_i . W_j . \left[[B1]^{T} \, [D1] \, [B1] + [B2]^{T} \, [D2] \, [B2] \right]$$

The element load vector corresponding to node i due to distributed load of intensity qz is obtained as -

$$\{fi\} = qz . \begin{Bmatrix} \int Ni . da \\ O \end{Bmatrix} = qz . \begin{Bmatrix} \Sigma \Sigma W_i . W_j . Ni . |J| \\ O \end{Bmatrix}$$

The discretisation of plate into finite elements, assembly of element stiffness matrices and solution of equations follow standard finite element procedure.

A computer program is prepared to incorporate these features.

4. NUMERICAL EXAMPLES

A homogeneous isotropic plate with all edges simply supported is considered for analysis. The symmetry of the plate about X and Y axes allows to analyse only quarter of the plate. The position and number of holes are selected such that the symmetry condition can not be violated for perforated plate. The Symmetry conditions are, (i) rotation of normal about X-axis Bx = 0, along X- axis and (ii) rotation of normal about Y- axis By = 0 along Y- axis. At simple support the boundary condition is w = 0 and free edge condition at the boundary of the hole.

Fig. 4.1 to fig. 4.4 shows the coordinate system, displacement, parent and mapped element, discretisation of solid plate in 5x5 mesh and typical finite element model of perforated plate.

The constant numerical data adopted for analysis of perforated plate is-

(Length of sq. plate) a = 200 cm, (Thickness of Plate) t = 1 cm, (Youngs modules) E = 2.1 x 106 Kg/cm2,

(Intensity of distributed load) qz = 0.1 Kg/cm2, (Possion's Ratio) ? = 0.3

Various numerical problems are solved. Result of analysis of solid square plate shows very close agreement of maximum deflection and maximum bending moment with the analytical solution. Square plate with central circular plate of radius ro = 50 cm and the plate with increasing the size of hole, thus increasing the percentage opening ? were analysed. Study of plate with number of square holes and combination of square, circular and rectangular holes are solved for different aspect ratios, (b/a, d/a) (b= Length of sq. hole and least lateral dimension of rectangular hole, d= diameter of circular hole)

The results of maximum deflection and moments are presented in tabular form, graphical form.

5. RESULTS AND DISCUSSION

From the results obtained for plate with central circular hole it is observed that the deflection obtained from the present analysis is greater than those obtained from the linear displacement finite element analysis of Seide and Change (9) and smaller than the deflection obtained from the finite element analysis using quadratic displacement triangular element with straight sides of Chaudhuri and Seide (7) and using EI - Hashmiy's (8) theory. It can be seen that the transverse displacement of the linear element due to Seide and Chang (9) are yet to converge to the correct solution, because this element is too stiff in bending. Also, Chaudhari and Seide (7) obtained the results using an assumed quadratic displacement element with straight sides. This element behave like a subparametric element and when this element version is used to model the curved boundary layer effect. It should be remembered that EI- Hashimy's solution have been obtained by satisfying the boundary conditions only at three points of the plate boundary and boundary of the hole. Therefore, a traction free edge condition is not satisfied in general. EI- Hashimy has neglected shear. Hence the present analysis gives the correct solution of the perforated plate.

6. CONCLUSION

The present work deals with the analysis of perforated plate by finite element method using Hinton's eight noded isoparametric plate element. This eight noded isoparametric plate element based on thick plate theory enhanced the applicability of finite element method for analysis of perforated plates. The maximum deflection and maximum bending moment for solid square plate shows very close agreement with the analytical solution (Fig. 4.5) which validate the present analysis and computer programme developed. The effect of various combination of hole and area of opening with respect to solid plate on the maximum deflection and moments is studied. Since the present analysis gives the correct solution of the perforated plate, it is possible to solve various problems related to perforated plates with different combination of perforations and boundary conditions and present the result in tabular and graphical forms which will increase the practical utility of the present work.

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