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APPLICATION OF TWO-DIMENSIONAL FRACTIONAL COSINE TRANSFORM TO DIFFERENTIAL EQUATION

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Abstract: Fractional cosine and sine transform are closely related to fractional Fourier transform which is most essential tool in the theory of optics and signal processing. Hence these transform are also used suitably in optics and signal processing as it reduces complexities of computation**.** The aim of this paper is we introduced new differential operator and also its ad joint operator

Keywords: Fractional Fourier Transform, Fractional Cosine Transform, Fractional Sine Transform.

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INTRODUCTION

Integral transforms provided a well establish and valuable method for solving initial and boundary value problem arising in several areas of physics, Applied Mathematics and Engineering. There are several works on the theory and application of integral transform such as the Laplace, Fourier, Mellin and Hankel etc. [2]. A.H. Zemanian [1] studied different integral transforms in the distributional generalized sense. The fractional integral transforms play an important role in signal processing. Fourier analysis is one of the most frequently used tools in signal processing and many other scientific disciplines.

Namias [4] introduced the concept of Fourier transform of fractional order, the fractional Fourier transform with corresponds to the classical Fourier transform and fractional Fourier transform with corresponds to the identity operator. The fractional Fourier transforms and its properties were discussed in Ozaktas [6]. Bhosale and Chaudhary [3] had extended it to the distribution of compact support using the eigenvalues function, as used in fractional Fourier transform, different integral transform in Fourier class that is cosine transform and sine transform, are generalized to fractional transform by Pei [5].

Bhosale [3] had given the application of generalized fractional Fourier transform for solving particular type of partial differential equation. He also introduce new operator and solve particular type of differential equation. In our previous work we have defined the following terms.

1.1. Generalized two dimensional fractional Cosine transform

Two dimensional fractional Cosine transform with parameter α f(x, y) denoted by $F_C^\alpha(x,y)$ perform a linear operation given by the integral transform.

$$F_C^\alpha\{f(x,y)\}(u,v) = \int_0^\infty \int_0^\infty f(x,y)K_\alpha(x,y,u,v)\,dx\,dy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .(1.1)$$

Where the kernel,

$$K_c^\alpha(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha . ux) . \cos(cosec\alpha . vy) \ldots\ldots\ldots (1.2)$$

1.2. The test function space E

An infinitely differentiable complex valued function $\emptyset$ on R^n belongs to $E(R^n)$ if for each copactset$I \subset S_{a,b}$, where,

$$S_{a,b} = \{x, y: x, y \in R^n, |x| \le a, |y| \le b, a > 0, b > 0\}, I \in R^n$$

$$\gamma_{E_{p,q}}(\emptyset) = \sup_{x,y}\left|D_{x,y}^{p,q}\emptyset(x,y)\right| < \infty \text{Where, p, q =1, 2, 3....}$$

Thus $E(R^n)$ will denote the space of all $\emptyset \in E(R^n)$ with support contained in $S_{a,b}$

Note: that the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member of E^*, the dual space of E.

In the present work, generalization of two dimensional fractional cosine transform is presented. The new adjoint operator is defined. And using it the differential equation is solved.

2. Distributional two-dimensional fractional Cosine transform

The two dimensional distributional fractional Cosine transform of $f(x, y) \in E^*(R^n)$ defined by

$$F_c^{\alpha}\{f(x, y)\} = F^{\alpha}(u, v) = \langle f(x, y), K_{\alpha}(x, y, u, v)\rangle \text{ .... } (2.1)$$

$$K_c^{\alpha}(x, y, u, v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\,.\cos(cosec\alpha.vy) \text{ ..... } (2.2)$$

Where , RHS of equation (2.1) has a meaning as the application of $f \in E^*$ to $K_{\alpha}(x, y, u, v) \in E$

3 Application of two dimensional fractional cosine transform.

Kernel of two dimensional fractional cosine transform as

$$K_c^{\alpha}(x, y, u, v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\,.\cos(cosec\alpha.vy).$$

We can arrange as,

$$K_c^{\alpha}(x, y, u, v) = A\, e^{\frac{i(x^2+y^2)cot\alpha}{2}} \cos(Px)\,.\cos(Qy)$$

Where $A = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2)cot\alpha}{2}}$ $\quad P = cosec\alpha.u \quad Q = cosec\alpha.v$

$$D_x D_y K_c^{\alpha}(x, y, u, v) = D_x D_y A\, e^{\frac{i(x^2+y^2)cot\alpha}{2}} \cos(Px)\,.\cos(Qy)$$

$$D_x D_y K_c^{\alpha}(x, y, u, v) = A D_x(e^{\frac{i(x^2)cot\alpha}{2}} \cos(Px)) D_y(e^{\frac{i(y^2)cot\alpha}{2}} \cos(Qy))$$

$$D_x D_y K_c^\alpha(x,y,u,v) = A\left[-PsinPxe^{\frac{i(x^2)cot\alpha}{2}} + ixcot\alpha.cosPxe^{\frac{i(x^2)cot\alpha}{2}}\right]$$

$$\left[-QsinQye^{\frac{i(y^2)cot\alpha}{2}} + iycot\alpha.cosQye^{\frac{i(y^2)cot\alpha}{2}}\right]$$

$$D_x D_y K_c^\alpha(x,y,u,v) = Ae^{\frac{i(x^2+y^2)cot\alpha}{2}}[-PsinPx + ixcot\alpha.cosPx][-QsinQy + iycot\alpha.cosQy]$$

$$D_x D_y K_c^\alpha(x,y,u,v) = Ae^{\frac{i(x^2+y^2)cot\alpha}{2}}cosPxcosQy[-PtanPx + ixcot\alpha][-QtanQy + iycot\alpha]$$

$$D_x D_y K_c^\alpha(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\cos(cosec\alpha.vy)$$

$$\begin{Bmatrix}[-cosec\alpha.u\,tan(cosec\alpha.ux) + ixcot\alpha] \\ [-cosec\alpha.vtan(cosec\alpha.vy) + iycot\alpha]\end{Bmatrix}$$

$$D_x D_y K_c^\alpha(x,y,u,v) = K_c^\alpha(x,y,u,v)\begin{Bmatrix}[-cosec\alpha.u\,tan(cosec\alpha.ux) + ixcot\alpha] \\ [-cosec\alpha.vtan(cosec\alpha.vy) + iycot\alpha]\end{Bmatrix}$$

$$\begin{aligned} D_x D_y K_c^\alpha(x,y,u,v) \\ &= K_c^\alpha(x,y,u,v)\{i^2xycot\alpha - ixcot\alpha csc\alpha.vtan(cosec\alpha.vy) \\ &- icosec\alpha.u\,tan(cosec\alpha.ux)ycot\alpha \\ &+ csc^2\alpha.u.v.tan(cosec\alpha.ux)tan(cosec\alpha.vy)\}\end{aligned}$$

$$\begin{aligned} D_x D_y K_c^\alpha(x,y,u,v) \\ &= K_c^\alpha(x,y,u,v)\{-xycot\alpha - ixcot\alpha csc\alpha.vtan(cosec\alpha.vy) \\ &- icosec\alpha.u\,tan(cosec\alpha.ux)ycot\alpha \\ &+ csc^2\alpha.u.v.tan(cosec\alpha.ux)tan(cosec\alpha.vy)\}\end{aligned}$$

$$\Lambda_{x,y} = x^{-1}y^{-1}D_xD_y - \begin{Bmatrix}\left(cot\alpha - \dfrac{icot\alpha csc\alpha.v.\tan(csc\alpha.vy)}{y}\right) \\ -\dfrac{icsc\alpha.u.ycot\alpha\tan(csc\alpha.ux)}{x} \\ +\dfrac{csc^2\alpha u.v\tan(csc\alpha.ux)\tan(csc\alpha.vy)}{xy}\end{Bmatrix}$$

$$\Lambda_{x,y}K_c^{\alpha}(x,y,u,v)$$
$$= x^{-1}y^{-1}D_xD_yK_c^{\alpha}(x,y,u,v)$$
$$-\left\{\begin{matrix}\left(cot\alpha - \frac{icot\alpha csc\alpha.v.\tan(csc\alpha.vy)}{y}\right) \\ -\frac{icsc\alpha.u.ycot\alpha\tan(csc\alpha.ux)}{x} \\ +\frac{csc^2\alpha u.v\tan(csc\alpha.ux)\tan(csc\alpha.vy)}{xy}\end{matrix}\right\}K_c^{\alpha}(x,y,u,v)$$

$$\Lambda_{x,y}K_c^{\alpha}(x,y,u,v)$$
$$= x^{-1}y^{-1}(K_c^{\alpha}(x,y,u,v)\{-xycot\alpha - ixcot\alpha csc\alpha.vtan(cosec\alpha.vy)$$
$$- icosec\alpha.u\, tan(cosec\alpha.ux)ycot\alpha$$
$$+ csc^2\alpha.u.v.tan(cosec\alpha.ux)tan(cosec\alpha.vy)\})$$
$$-\left\{\begin{matrix}\left(cot\alpha - \frac{icot\alpha csc\alpha.v.\tan(csc\alpha.vy)}{y}\right) \\ -\frac{icsc\alpha.u.ycot\alpha\tan(csc\alpha.ux)}{x} \\ +\frac{csc^2\alpha u.v\tan(csc\alpha.ux)\tan(csc\alpha.vy)}{xy}\end{matrix}\right\}K_c^{\alpha}(x,y,u,v)$$

$$\Lambda_{x,y}K_c^{\alpha}(x,y,u,v)$$
$$= \left\{\begin{matrix} x^{-1}y^{-1}\left\{\begin{matrix}-xycot\alpha - ixcot\alpha csc\alpha.vtan(cosec\alpha.vy) \\ -icosec\alpha.u\, tan(cosec\alpha.ux)ycot\alpha \\ +csc^2\alpha.u.v.tan(cosec\alpha.ux)tan(cosec\alpha.vy)\end{matrix}\right\} \\ -\left\{\begin{matrix}\left(cot\alpha - \frac{icot\alpha csc\alpha.v.\tan(csc\alpha.vy)}{y}\right) \\ -\frac{icsc\alpha.u.ycot\alpha\tan(csc\alpha.ux)}{x} \\ +\frac{csc^2\alpha u.v\tan(csc\alpha.ux)\tan(csc\alpha.vy)}{xy}\end{matrix}\right\}\end{matrix}\right\}K_c^{\alpha}(x,y,u,v)$$

$$\wedge_{x,y} K_c^{\alpha}(x,y,u,v)$$
$$= \left\{ -cot\alpha - \frac{icot\alpha csc\alpha . vtan(cosec\alpha . vy)}{y} - \frac{icosec\alpha . u \, tan(cosec\alpha . ux) ycot\alpha}{x} + \frac{csc^2\alpha . u . v . tan(cosec\alpha . ux) tan(cosec\alpha . vy)}{xy} - cot\alpha + \frac{icot\alpha csc\alpha . v . \tan(csc\alpha . vy)}{y} + \frac{icsc\alpha . u . ycot\alpha \tan(csc\alpha . ux)}{x} - \frac{csc^2\alpha u . v\tan(csc\alpha . ux)\tan(csc\alpha . vy)}{xy} \right\} K_c^{\alpha}(x,y,u,v)$$

$$\wedge_{x,y} K_c^{\alpha}(x,y,u,v) = \{-2cot\alpha\} K_c^{\alpha}(x,y,u,v)$$

$$\wedge_{x,y} K_c^{\alpha}(x,y,u,v) = \{2i^2 cot\alpha\} K_c^{\alpha}(x,y,u,v)$$

$$\wedge_{x,y}{}^2 K_c^{\alpha}(x,y,u,v) = (2i^2 cot\alpha)^2 K_c^{\alpha}(x,y,u,v)$$

$$\wedge_{x,y}{}^2 K_c^{\alpha}(x,y,u,v) = (C_\alpha)^2 K_c^{\alpha}(x,y,u,v)$$

Where $C_\alpha = 2i^2 cot\alpha$

$$\wedge_{x,y}{}^3 K_c^{\alpha}(x,y,u,v) = (C_\alpha)^3 K_c^{\alpha}(x,y,u,v)$$

$$\wedge_{x,y}{}^4 K_c^{\alpha}(x,y,u,v) = (C_\alpha)^4 K_c^{\alpha}(x,y,u,v)$$

So on

$$\wedge_{x,y}{}^k K_c^{\alpha}(x,y,u,v) = (C_\alpha)^k K_c^{\alpha}(x,y,u,v)$$

Therefore we have

$$F_\alpha^c\{\wedge_{x,y}^k f(x,y)\} = \langle f(x,y), (C_\alpha)^k K_c^{\alpha}(x,y,u,v)\rangle$$

Foe all $f \epsilon E'$ and for $0 < \alpha < \frac{\pi}{2}$

4. Ad joint Operator $\wedge_{x,y}^*$

We define an operator $\wedge_{x,y}^* : \boldsymbol{E}' \to \boldsymbol{E}$ using the relation

$\langle \wedge_{x,y}^* \{f(x,y)\}, \varphi(x,y)\rangle = \langle f(x,y), \wedge_{x,y} \varphi(x,y)\rangle$ For all $f \epsilon E'$ and $\varphi \; \epsilon E$

The operator $\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*}$ is called the ad joint operator of $\wedge_{x,y}$ for each k=1, 2, 3…

We easily get

$\langle (\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}}\{f(x,y)\}, \varphi(x,y)\rangle = \langle f(x,y), (\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}}\varphi(x,y)\rangle$

It can be readily shown that f is regular distribution generated by an element in then

$\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*}\boldsymbol{f} = \wedge_{x,y} f$

For each k=1, 2, 3…… and for $0 < \alpha < \frac{\pi}{2}$ we have

$\langle (\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}}\{f(x,y)\}, K_c^{\alpha}(x,y,u,v)\rangle = \langle f(x,y), (\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}} K_c^{\alpha}(x,y,u,v)\rangle$

$\langle (\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}}\{f(x,y)\}, K_c^{\alpha}(x,y,u,v)\ \rangle = \langle f(x,y), (2i^2 cot\alpha)^{\boldsymbol{k}} K_c^{\alpha}(x,y,u,v))\rangle$

$\langle (\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}}\{f(x,y)\}, K_c^{\alpha}(x,y,u,v)\ \rangle = \langle f(x,y), (C_{\alpha})^{\boldsymbol{k}} K_c^{\alpha}(x,y,u,v))\rangle$

$\langle (\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}}\{f(x,y)\}, K_c^{\alpha}(x,y,u,v)\ \rangle = (C_{\alpha})^{\boldsymbol{k}}\langle f(x,y), K_c^{\alpha}(x,y,u,v))\rangle$

Thus we arrive at the important result, for each k=1, 2, 3….. And for $0 < \alpha < \frac{\pi}{2}$

We have for $f\epsilon E'$

$$F_{\alpha}^{c}\left\{(\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})^{\boldsymbol{k}}\{f(x,y)\}\right\} = (C_{\alpha})^{\boldsymbol{k}} F_{\alpha}^{c}\{\{f(x,y)\}\}(u,v)$$

5. An application of the two dimensional fractional cosine transform to differential equations of

$\boldsymbol{P}(\wedge_{\boldsymbol{x},\boldsymbol{y}}^{*})\boldsymbol{U} = \boldsymbol{f}$

Solution: consider the differential equation $P(\wedge_{x,y}^{*})U = f$……………. (1)

where $f\epsilon E'$ and P any polynomial of degree m.

Suppose that the equation (1) possesses a solution U.

Applying the fractional cosine transform to (1)

We have,

$F_\alpha^c\left[P\left(\wedge_{x,y}^*\right)U\right] = F_\alpha^c(f(x,y))$ If $F_\alpha^c(f) = f^\wedge$ then

$P(2i^2 cot\alpha)F_\alpha^c\big(f(x,y)\big) = f^\wedge$ Using

$F_\alpha^c\left\{\left(\wedge_{x,y}^*\right)^{\boldsymbol{k}}\{f(x,y)\}\right\} = (2i^2 cot\alpha)^{\boldsymbol{k}} F_\alpha^c\{\{f(x,y)\}\}$

$P(2i^2 cot\alpha)U^\wedge = f^\wedge$

$P(C_\alpha)U^\wedge = f^\wedge$ ………………………………………………………… (2)

Where $U^\wedge = F_\alpha^c\{\{U(x,y)\}\}$

If we further assume that the polynomial P is such that for $\epsilon > 0$

$|P(2i^2 cot\alpha)| <\in$ For $0 < \alpha \leq \frac{\pi}{2}$

Then under this assumption (2) gives

$U^\wedge = [P(2i^2 cot\alpha)]^{-1} f^\wedge$

Applying the inversion of fractional cosine transform we get

$U = (F_\alpha^c)^{-1}\left[\frac{f^\wedge}{P(2i^2 cot\alpha)}\right] = (F_\alpha^c)^{-1}\left[\frac{f^\wedge}{P(C_\alpha)}\right]$

Hence proof.

Conclusion: In the present work, the new adjoint operator for two dimensional fractional cosine transform is defined. Using it the differential equation is solved.

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