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REDUCTION TECHNIQUES IN DYNAMIC ANALYSIS OF STRUCTURES

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Abstract: For stress and deformation analysis of tall structures, bridges, ships, aircraft, automobiles, nuclear reactors and many other structures, large finite element models with thousands of degrees of freedom (dof) are used. Use of same detailed representation that is required for static analysis is not practically possible while performing the dynamic analyses. Moreover it becomes unnecessary also and the entire project becomes tremendously cumbersome. Furthermore, design and control methods work best for the systems with a smaller number of degrees of freedom. To overcome this difficulty dynamic reduction techniques have been developed to reduce the number of degrees of freedom, prior to performing dynamic analysis. If dynamic reduction is not used the solution cost will dominate for the practical industrial problems.

Keywords: Stress and Deformation Analysis, Dynamic Analyses, Degrees Of Freedom, Dynamic Reduction Techniques.

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INTRODUCTION

In general while performing dynamic analysis of structures using FEM, sometimes it becomes necessary to discretize a structure into a large number of elements because of changes in geometry, loading or material properties. However it is not always possible to obtain rigorous mathematical solutions for the entire structure. While assembling the elements for entire structure, the number of unknown displacements, that is, the number of degrees of freedom may be quite large. As a result of which the stiffness, mass and damping matrices will be of large size. The solution of corresponding Eigen problem to compute natural frequencies and modal shapes will be difficult and moreover expensive. In such cases it is advisable to reduce the size of these matrices in order to make the solution of eigen problem more manageable and economical.

Four major steps in dynamic analysis are

1. Assembly of equations
2. Dynamic reduction (optional)
3. Solution of equations
4. Recovery of forces, stresses.

If dynamic reduction is not used the solution cost will dominate for the practical industrial problems.

II. REVIEW OF LITERATURE

FEM based dynamic analysis seldom gives correct results matching with the experimental behavior of the structure. The research on the model updating is yet to have a reliable method which suits for present industrial requirement. All the methods lack in generality and useful on case specific basis. List of the publications is exhaustive and lot of literature is available. An attempt is made to highlight few techniques and latest development in the model updating. Many methods of reducing dynamic matrices used in FEM analysis are currently in use. Out of them following three are more popular.

1. Guyan reduction method,
2. Improved reduction system and
3. Dynamic Reduction system

The simplest procedure of Guyan [1] assumes that the interpolation shapes can be calculated using the FEM stiffness and/or mass matrices to solve a static solution.

The Improved Reduction System (IRS) method developed by O'Callahan [2] improves upon Guyan reduction by including a first order estimate for mass effects in the development of Transformation Matrix.

Another approach to improve on Guyan reduction is called Dynamic Reduction system proposed by Paz [3]

Friswell et. al. [4] and Freed et.al.[5] have presented additional discussion of TAM (Test Analysis Models) reduction and expansion.

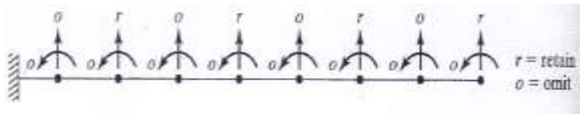
IRS reduction method can be easily performed in NASTRAN using a rigid format alter [6, 7]. The weaknesses of IRS reduction method were shown by Gordis [8].

However, these more advanced methods can encounter numerical problems if the accelerometer set is inadequate.

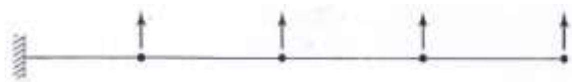
In general, Guyan reduction should be used due to its simplicity and greater experience base. In this paper this method is re-mastered and a computer program has been developed which gives a sufficiently closer dynamic behavior of the structure

III. FORMULATION

Consider a 16 dof model of a cantilever beam as shown in Fig.1(a). We have to make the decision as to which dof are to be retained and which are to be omitted. The retained dof may be termed as master dof and the omitted ones may be slaves. Fig.1(b) shows how a reduced model is obtained by omitting certain dof. The omitted dof correspond to those at which the applied and inertial forces are negligible.



(a) 16 dof model



(b) 4 dof model

Fig. 1. FEM models

The reduced stiffness and mass matrices are obtained as follows

The equations of motion are :

$$\mathbf{M\,\ddot{u} + K\,u = F} \quad \ldots (1)$$

If inertial forces and applied forces are grouped together, the we can write the equations as :

$$\mathbf{K\,u = F} \quad \ldots (2)$$

The displacement set **u** can be partitioned as $\mathbf{u_r}$ = retained set and $\mathbf{u_o}$ = omitted set.

$$\mathbf{u} = \begin{Bmatrix} \mathbf{u_r} \\ \mathbf{u_o} \end{Bmatrix} \quad \ldots (3)$$

To start with the retained set may be taken as 20 % of the total dof . The equations of motion can now be written in partitioned form as :

$$\begin{bmatrix} \mathbf{K_{rr}} & \mathbf{K_{ro}} \\ \mathbf{K_{ro}^T} & \mathbf{K_{oo}} \end{bmatrix} \begin{Bmatrix} \mathbf{u_r} \\ \mathbf{u_o} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F_r} \\ \mathbf{F_o} \end{Bmatrix} \quad \ldots (4)$$

The idea is to choose the omitted set such that the components of $\mathbf{F_O}$ are small . Thus we should retain those dof in the r – set with large and concentrated masses which are loaded and which are needed to adequately describe the mode shape. Setting $\mathbf{F_O = 0}$ **,** the lower part of equation (4) yields :

$$\begin{bmatrix} \mathbf{K_{rr}} & \mathbf{K_{ro}} \\ \mathbf{K_{ro}}^{\mathbf{T}} & \mathbf{K_{oo}} \end{bmatrix} \begin{Bmatrix} \mathbf{u_r} \\ \mathbf{u_o} \end{Bmatrix} = \begin{Bmatrix} \mathbf{Fr} \\ \mathbf{0} \end{Bmatrix} \quad \ldots (5)$$

Solving for the omitted dof

$[\mathbf{K_{rr}}]\{\mathbf{u_r}\} + [\mathbf{K_{oo}}]\{\mathbf{u_o}\} = 0$

$\{\mathbf{u_o}\} = -[\mathbf{K_{oo}}]^{-1}[\mathbf{K_{or}}]\{\mathbf{u_r}\}$ ... (6)

The strain energy in the structure is

$\mathbf{U} = \frac{1}{2}\ \mathbf{u^T K u}$ and can be written as

$$\mathbf{U} = \frac{1}{2}\,[\mathbf{u_r}^T\ \mathbf{u_o}^T]\begin{bmatrix}\mathbf{K_{rr}} & \mathbf{K_{ro}} \\ \mathbf{K_{ro}}^T & \mathbf{K_{oo}}\end{bmatrix}\begin{Bmatrix}\mathbf{u_r} \\ \mathbf{u_o}\end{Bmatrix}$$

Substituting eqn.(6) into the above, we can write $\mathbf{U} = \frac{1}{2}\ \mathbf{u_r}^T \mathbf{K_r}\,\mathbf{u_r}$, where $\mathbf{K_r}$ is the reduced stiffness matrix given by :

$\mathbf{K_r = K_{rr} - K_{ro}\,K_{oo}^{-1}\,K_{ro}^T}$ ... (7)

To obtain an expression for reduced mass matrix, we consider the kinetic energy

$V = \frac{1}{2}\ \dot{u}^T \mathbf{M}\,\dot{u}$. Upon partitioning the mass matrix and using eqn. (6), we can write the kinetic energy as $\mathbf{V} = \frac{1}{2}\ \dot{\mathbf{u}}_\mathbf{r}^T \mathbf{M_r}\,\dot{\mathbf{u}}_\mathbf{r}$ **,** where $\mathbf{M_r}$ is the reduced matrix given by

$$\mathbf{M_r = M_{rr} - M_{ro}\,K_{oo}^{-1}\,K_{ro}^T - K_{ro}Koo^{-1}\,Mro^T}$$

$$\mathbf{+\ K_{ro}K_{oo}^{-1}\,Moo\,K_{oo}^{-1}\,K_{ro}^T} \quad ...(8)$$

Now, with these reduced stiffness and mass matrices, only a smaller eigenvalue problem is required to be solved.

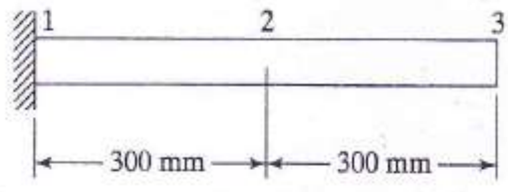
$\mathbf{K_r\,U_r = \lambda\,M_r\,U_r}$ ... (9)

Then we recover,

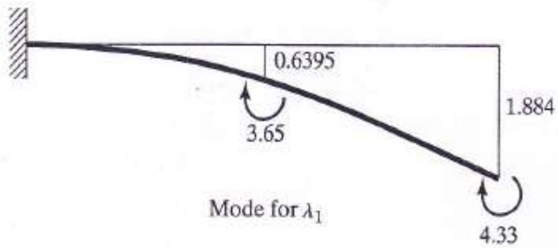
$\mathbf{U_o = -\,K_{oo}^{-1}\,K_{ro}^T\ U_r}$ ... (10)

IV NUMERICAL EXAMPLES

For establishing the validity of program, following example of a cantilever is considered.



(a)



(b)

E = 200 GPa, ρ = 7840 kg/m^3,

I = 2000 mm^4, A = 240 mm^2

Fig. 2. Cantilever beam model

Example 1: A cantilever having geometrical and material properties as shown in fig. 2 is analysed for modal analysis. The degree of freedom at each node is deflection (amplitude) w and slope $\partial w/\partial x$. Using standard finite element procedure [9], we obtain the stiffness and mass matrices by hand calculations:

$$\mathbf{K} = 1000\begin{bmatrix} 355.6 & 0 & -177.8 & 26.67 \\ 0 & 10.67 & -26.67 & 2.667 \\ -177.8 & -26.67 & 177.8 & -26.67 \\ 26.67 & 2.667 & -26.67 & 5.33 \end{bmatrix}$$

$$\mathbf{M} = 0.001\begin{bmatrix} 419.3 & 0 & 72.6 & -5.20 \\ 0 & 0.967 & 5.20 & -0.36 \\ 72.6 & 5.20 & 209.7 & -8.90 \\ -5.20 & -0.36 & -8.90 & 0.48 \end{bmatrix}$$

Now evaluate the lowest two eigenvalues and corresponding eigenmodes for the beam based on a 4 dof model using Guyan's reduction method.

Solution : For applying Guyan's reduction method to this problem based on omitting the rotational dof and comparing the results with the full model, refer fig.2 u_3, u_5 refer to translational dof while u_4, u_6 refer to the rotational dof. Thus the retained set is u_3, u_5 and the omitted set is u_4, u_6. Extracting the appropriate components from the full 4 x 4 **K** and **M** matrices we obtain

$$\mathbf{k}_{rr} = 1000 \begin{bmatrix} 355.6 & -177.8 \\ -177.8 & 177.8 \end{bmatrix}$$

$$\mathbf{k}_{ro} = 1000 \begin{bmatrix} 0 & 26.67 \\ -26.67 & -26.67 \end{bmatrix}$$

$$\mathbf{k}_{oo} = 1000 \begin{bmatrix} 10.67 & 2.667 \\ 2.667 & 5.33 \end{bmatrix}$$

$$\mathbf{m}_{rr} = \begin{bmatrix} 0.4193 & 0.0726 \\ 0.0726 & 0.2097 \end{bmatrix}$$

$$\mathbf{m}_{ro} = \begin{bmatrix} 0 & -0.0052 \\ 0.0052 & -0.0089 \end{bmatrix}$$

$$\mathbf{m}_{oo} = \begin{bmatrix} 0.000967 & -0.00036 \\ -0.00036 & 0.00048 \end{bmatrix}$$

From equations (7) and (8), we obtain the reduced matrices

$$\mathbf{K}_r = 10000 \begin{bmatrix} 20.31 & -6.338 \\ -6.338 & 2.531 \end{bmatrix},$$

$$\mathbf{M}_r = \begin{bmatrix} 0.502 & 0.1 \\ 0.1 & 0.155 \end{bmatrix}$$

An input data file is prepared and the program JACOBI2 is used to solve the eigenvalue problem in eq(9). The solution is

$\lambda_1 = 20250$, $\lambda_2 = 818300$

$U_r^1 = \begin{Bmatrix} 0.6395 \\ 1.884 \end{Bmatrix}$, $U_r^2 = \begin{Bmatrix} 1.370 \\ -1.959 \end{Bmatrix}$

Using eqn. (10), we obtain the eigenvector components corresponding to the omitted dof as

$U_o^1 = \begin{Bmatrix} 3.65 \\ 4.33 \end{Bmatrix}$, $U_o^2 = \begin{Bmatrix} -0.838 \\ -16.238 \end{Bmatrix}$

The results of analysis are summarized in table 1. All the results show good agreement with the results reported in literature [10,11].

Table 1

Results reported in literature in full form		**Results obtained with present analysis**	
Natural frequency (eigen value) λ	Mode shapes (eigen vectors) U	Natural frequency (eigen value) λ	Mode shapes (eigen vectors) U
2.0304×10^4	0.64, 3.65, 1.88, 4.32	2.025×10^4	0.6395, 3.65 1.884 4.33
8.0987×10^5	-1.37,	8.183×10^5	-1.375,

	1.39,	1.386,
	1.901,	1.906,
	15.27	15.27

V. CONCLUDING REMARKS

In this paper, the application of various reduction techniques was studied with their prominent features and computer programs were developed to get dynamic behaviour of structures. Following hints shall be followed while using dynamic reduction techniques.

- Identify dof to be retained in the analysis set and distribute it uniformly over the structure.
- Put the points with large masses in a set.
- It is better for the retained set to have both translations and rotations in case of structural elements.

Natural frequencies and mode shapes give us the needed data concerning what excitation frequencies should be avoided.

REFERENCES:

1. Guyan R. J.,“ Reduction of Stiffness and Mass Matrices”, AIAA Journal, Vol. 3, February, 1965.

2. O’Callahan J., “A Procedure for an Improved Reduced System (IRS) Model”, 7th International Modal Analysis Conference, January, 1989.

3. Paz Mario, “Structural Dynamics Theory and Computation”, CBS Publishers & Distributors Pvt. Ltd. ISBN: 81-239-0978-0, 2nd Edition, 2004, pp 247-277.

4. Friswell M.I. and Mottershead J. E., “Finite Element Model Updating in Structural Dynamics”, Kluwer academic Publishers, ISBN 0-7923-3431-0, 1995.

5. Freed A.M. and Flanigan C.C., “A comparison of Test –Analysis Model Reduction Methods”, 8th International Modal Analysis Conference, January, 1990.

6. Flanigan C., “Implementation of the IRS Dynamic Reduction Method in MSC/NASTRAN”, MSC/NASTRAN World Users Conference, Los Angeles, CA, March 1990.

7. Flanigan C., "Development of the IRS Component Dynamic Reduction Method for substructure Analysis", AIAA-91-1056, AIAA 32nd Structures, Structural Dynamics and Materials Conference, April 1991.

8. Gordis J.H., "An Analysis of Improved Reduced System (IRS) Model Reduction Procedure", 10th International Modal Analysis Conference, February, 1992.

9. A.S. Meghre & K.N. Kadam – Finite Element Method in Structural Analysis – First Edition – 2014

10. T.R.Chandrupatla and A.D. Belegundu – Introduction to Finite Elements in Engineering – Second Edition Prentice Hall of India Pvt. Ltd. 1998

11. Klaus-Jurgen Bathe- Finite Element Procedures – Fourth Reprint, Prentice Hall of India Pvt. Ltd. 1997.