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BIANCHI TYPE IX COSMOLOGICAL MODEL USING TWO FLUIDS WITH MAGNETIC FLUX IN GENERAL RELATIVITY

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Abstract: In this paper, we have studied the nature of Bianchi type-IX cosmological model with two fluids i.e. with matter and radiating source in presence of electromagnetic field in general relativity. In this model, the fluid represents, $T_j^{i\,m}$is one of the matter contents of the universe and another fluid $T_j^{i\,r}$ is the CMB radiation. The magnetic field is due to an electric current produced along x-axis, F_{23} is the only non-vanishing component of electromagnetic field tensorF_{ij}. To get the deterministic solution, it has been assumed that the relation, $a = b^n$ in which a and b are metric potential and n is constant. The physical and geometrical properties of this model were discussed.

Keywords: Bianchi type-IX space-time, Two fluids, CMB radiation, Electromagnetic field, magnetic permeability.

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INTRODUCTION

The Bianchi type cosmological models are important in the sense that, these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time, Moreover , from the theoretical point of view anisotropic universe has a greater generality than isotropic models. The Bianchi type IX cosmological models are interesting to study because these models allow not only expansion but also rotation and shear, and in general these are anisotropic. Many relativists have taken keen interest in studying Bianchi type IX universe because familiar solutions like Robertson-Walker universe, the de Sitter universe, theTaub-Nut solutions etc. including closed FRW models. Vaidya and Patel [1] have studied spatially homogeneous space time of Bianchi type IX, and they have outlined a general scheme forth derivation of exact solutions of Einstein's field equations in the presence of perfect fluid and pure radiation fields. Bali and Yadav [2] have investigated Bianchi type IX viscous fluid cosmological models. Pradhan et al. [3] derived Bianchi type IX viscous fluid cosmological models with a varying cosmological constant. The dynamical effects of spatially homogeneous electromagnetic field on anisotropic Bianchi type IX models are studied by Waller [4]. Bali and Dave [5] have investigated Bianchi type IX string cosmological model in General Relativity. Tyagi et al. [6] have studied homogeneous Anisotropic Bianchi type cosmological model for perfect fluid distribution and electromagnetic field tensor.

The cosmic microwave background (CMB) is one of the cornerstones of the homogeneous, isotropic model. Anisotropies in the CMB are related to small perturbation, superimposed on the perfectly smooth background, which are believed to seed formation of galaxies and large scale structure in the universe. Coley and Dunn [7] have studied Bianchi type VI_0 model with two fluid sources. Pant and Oli [8] has been investigated two fluid cosmological models using Bianchi type II space-time. Oli [9] has presented anisotropic, homogeneous two fluid cosmological models in a Bianchi type I space time with a variable gravitational constant G and cosmological constant. Recently Adhav et al. [10] examined two fluid cosmological models in Bianchi type-V space-time. Also, Pawar et al. [11] has obtained Bianchi type IX with two fluid cosmological models. Recently, Patil et al. [12, 13] has studied Non shearing LRS Bianchi type III and IX string cosmological model in presence of magnetic flux with bulk viscosity.

In this paper, we have investigated Bianchi type IX cosmological model in presence of two fluid distributions with electro-magnetic field. By assuming F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get a deterministic model, we have assumed the condition $a = b^n$ between metric potentials, in which 'n' is constant. The physical and geometrical aspects of the models are also discussed.

2. FORMATIONS OF FIELD EQUATIONS:

We have consider homogeneous anisotropic Bianchi type IX metric,

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + (b^2 sin^2 y + a^2 cos^2 y)dz^2 - 2a^2 cosy\, dx\, dz \quad (1)$$

Where metric potentials a and b are functions of 't' alone.

The energy momentum tensor for two fluids source and electromagnetic field tensor is taken in the form,

$$T_j^i = {T_j^i}^m + {T_j^i}^r + E_j^i \quad (2)$$

Where, $${T_j^i}^m = (p_m + \rho_m)(u^i)^m (u_j)^m - p_m g_j^i \quad (2a)$$

And $${T_j^i}^r = \frac{4}{3}\rho_r (u^i)^r (u_j)^r - \frac{1}{3}\rho_r g_j^i \quad (2b)$$

The commoving coordinates system, for the line element (1) is ,

$${u_i}^m = (0,0,0,1), \ {u_i}^r = (0,0,0,1) ,$$

The electromagnetic field E_j^i is defined as,

$$E_j^i = \frac{1}{4\pi}\left[-F_{jl}F^{il} + \frac{1}{4}g_j^i F_{lm}F^{lm}\right] \quad (3)$$

The Maxwell's equation is,

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0 \quad (4)$$

Equation (4) leads to ,

$$F_{23} = H\, siny \quad (5)$$

The Einstein's field equation is,

$$R_j^i - \frac{1}{2}R g_j^i = -8\pi T_j^i \quad (6)$$

From equation (6), using equation (2), (3) and (1), we obtain,

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{1}{4}\frac{a^2}{b^4} = 8\pi\left(2p_m + \rho_m + \frac{5}{3}\rho_r\right) - \frac{H^2}{b^4} \quad (7)$$

$$2\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+\frac{1}{b^2}-\frac{3}{4}\frac{a^2}{b^4}=8\pi\left(p_m+\frac{1}{3}\rho_m\right)-\frac{H^2}{b^4} \tag{8}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}}{a}\frac{\dot{b}}{b}+\frac{1}{4}\frac{a^2}{b^4}=8\pi\left(p_m+\frac{1}{3}\rho_m\right)+\frac{H^2}{b^4} \tag{9}$$

$$\left(p_m+\frac{1}{3}\rho_m\right)=0 \tag{10}$$

To obtain more general solution, we use the metric potential relation $a = b^n$ (11)

And using equation (10) and (11) in equation (9), we obtain

$$\frac{db}{dt}=\dot{b} = \left(\frac{2H^2b^{-2}}{2n^2-2n-1}+\frac{b^{2n-2}}{2(1-2n^2)}\right)^{1/2} \tag{12}$$

Substitute $b = T$ (13)

The line element (1) takes the form,

$$ds^2 = -\left[\frac{2H^2T^{-2}}{2n^2-2n-1}+\frac{T^{2n-2}}{2(1-2n^2)}\right]^{-1} dT^2 + T^{2n}(t)dx^2 + T^2(t)dy^2$$
$$+(T^2sin^2y + T^{2n}cos^2y)dz^2 - 2T^{2n}cosy\, dx\, dz\,. \tag{14}$$

From the equation of state of matter, we have,

$$p_m = (\gamma-1)\rho_m \quad , \quad 1 \le \gamma \le 2\,.$$

To obtain, pressure, energy density of matter, energy density of radiation and total energy density by using in equation (7) and (8) and (10), we get

$$p_m 8\pi = \frac{(\gamma-1)}{(4-3\gamma)}\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+ H^2T^{-4}\right\} \tag{15}$$

$$\rho_r 8\pi = \frac{3(\gamma-1)}{(3\gamma-4)}\left[\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+ H^2T^{-4}\right] \tag{16}$$

$$\rho_m 8\pi = \frac{1}{(4-3\gamma)}\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+ H^2T^{-4}\right\} \tag{17}$$

As $\rho = \rho_r + \rho_m$

$$\rho = \frac{1}{8\pi}\left[\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+ H^2T^{-4}\right] \tag{18}$$

$$\theta = \frac{(n+2)}{T}\left[\frac{2H^2T^{-2}}{2n^2-2n-1} + \frac{T^{2n-2}}{2(1-2n^2)}\right]^{1/2} \tag{19}$$

$$\theta^2 = (n+2)^2\left[\frac{2H^2T^{-2}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right] \tag{20}$$

And,

$$\sigma^2 = \frac{1}{2}\left((n+2)^2\right)\left[\frac{2H^2T^{-2}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right] \tag{21}$$

Case I: Dust model when $\gamma = 1$

The scalar of expansion, shear scalar and deceleration parameter are obtained as,

$$\theta = \frac{(n+2)}{T}\left[\frac{2H^2T^{-2}}{2n^2-2n-1} + \frac{T^{2n-2}}{2(1-2n^2)}\right]^{1/2}$$

$$\sigma^2 = \frac{1}{2}\left((n+2)^2\right)\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right]$$

$$q = \frac{(1-n)\left(\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right) + \frac{3}{2}\left[\frac{2H^2T^{-3}}{2n^2-2n-1} + \frac{(n-1)T^{2n-3}}{2(1-2n^2)}\right]\left[\frac{2H^2}{2n^2-2n-1} + \frac{T^{2(n+1)}}{(1-2n^2)}\right]^{-1/2}}{(n+2)^3\left(\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right)^2}$$

From equations (16) and (17) yields,

$$\rho_r = 0$$

$$\rho_m = \frac{1}{8\pi}\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1) + \frac{1}{T^2} - \frac{1}{4}T^{2n-4} + H^2T^{-4}\right\}$$

$$\Omega_r = 0$$

$$\Omega_m = \frac{\frac{1}{8\pi}\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1) + \frac{1}{T^2} - \frac{1}{4}T^{2n-4} + H^2T^{-4}\right\}}{\frac{1}{3}(n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

$$\Omega = \Omega_m + \Omega_r$$

$$\Omega = \frac{\frac{1}{8\pi}\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1) + \frac{1}{T^2} - \frac{1}{4}T^{2n-4} + H^2T^{-4}\right\}}{\frac{1}{3}(n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

Case II: Radiation universe, when $\gamma = \frac{4}{3}$

The scalar of expansion, shear scalar and deceleration parameter are obtained as,

$$\theta = \frac{(n+2)}{T}\left[\frac{2H^2T^{-2}}{2n^2-2n-1} + \frac{T^{2n-2}}{2(1-2n^2)}\right]^{1/2}$$

$$\sigma^2 = \frac{1}{2}\left((n+2)^2\right)\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right]$$

$$q = \frac{(1-n)\left(\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right)+\frac{3}{2}\left[\frac{2H^2T^{-3}}{2n^2-2n-1}+\frac{(n-1)T^{2n-3}}{2(1-2n^2)}\right]\left[\frac{2H^2}{2n^2-2n-1}+\frac{T^{2(n+1)}}{(1-2n^2)}\right]^{-1/2}}{(n+2)^3\left(\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right)^2}$$

From equations (16) and (17) yields,

$$\rho_m \longrightarrow \infty\ ,\ \rho_r \longrightarrow \infty$$

$$\Omega_m \longrightarrow \infty,\ \Omega_r \longrightarrow \infty\ ,\Omega \longrightarrow \infty$$

Case III: Hard universe, when$\in \left(\frac{4}{3}\ ,\ 2\right)$, let us take $\gamma = \frac{5}{3}$,

The scalar of expansion, shear scalar and deceleration parameter are as follows,

$$\theta = \frac{(n+2)}{T}\left[\frac{2H^2T^{-2}}{2n^2-2n-1} + \frac{T^{2n-2}}{2(1-2n^2)}\right]^{1/2}$$

$$\sigma^2 = \frac{1}{2}\left((n+2)^2\right)\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right]$$

$$q = \frac{(1-n)\left(\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right)+\frac{3}{2}\left[\frac{2H^2T^{-3}}{2n^2-2n-1}+\frac{(n-1)T^{2n-3}}{2(1-2n^2)}\right]\left[\frac{2H^2}{2n^2-2n-1}+\frac{T^{2(n+1)}}{(1-2n^2)}\right]^{-1/2}}{(n+2)^3\left(\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right)^2}$$

From equations (16) and (17) we have obtained,

$$\rho_r = \frac{2}{8\pi}\left[\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1) + \frac{1}{T^2} - \frac{1}{4}T^{2n-4} + H^2T^{-4}\right]$$

$$\rho_m = -\frac{1}{8\pi}\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1} + \frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1) + \frac{1}{T^2} - \frac{1}{4}T^{2n-4} + H^2T^{-4}\right\}$$

$$\Omega_r = \frac{6\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right\}}{8\pi\ (n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

$$\Omega_m = \frac{-3\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right\}}{8\pi\ (n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

$$\Omega = \Omega_r + \Omega_m = \frac{3\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right\}}{8\pi\ (n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

Case IV: Zeldovich universe, when $\gamma = 1$

The scalar of expansion, shear scalar and deceleration parameter are given by

$$\theta = \frac{(n+2)}{T}\left[\frac{2H^2T^{-2}}{2n^2-2n-1}+\frac{T^{2n-2}}{2(1-2n^2)}\right]^{1/2}$$

$$\sigma^2 = \frac{1}{2}\left((n+2)^2\right)\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right]$$

$$q = \frac{(1-n)\left(\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right)+\frac{3}{2}\left[\frac{2H^2T^{-3}}{2n^2-2n-1}+\frac{(n-1)T^{2n-3}}{2(1-2n^2)}\right]\left[\frac{2H^2}{2n^2-2n-1}+\frac{T^{2(n+1)}}{(1-2n^2)}\right]^{-1/2}}{(n+2)^3\left(\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right)^2}$$

From equations (16) and (17) we obtained ,

$$\rho_r = \frac{3}{16\pi}\left[\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right]$$

$$\rho_m = -\frac{1}{16\pi}\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right\}$$

$$\Omega_r = \frac{9\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right\}}{16\pi\ (n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

$$\Omega_m = \frac{-3\left\{\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right\}}{16\pi\ (n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

$$\Omega = \frac{3\left\{\left[\frac{2H^2}{2n^2-2n-1}+\frac{T^{2n}}{2(1-2n^2)}\right](2n+1)+\frac{1}{T^2}-\frac{1}{4}T^{2n-4}+H^2T^{-4}\right\}}{8\pi\ (n+2)^2\left[\frac{2H^2T^{-4}}{2n^2-2n-1}+\frac{T^{2n-4}}{2(1-2n^2)}\right]}$$

4. CONCLUSION:

From above discussion, it is observed that, the model starts to expand with the big bang at $T = 0$ and stops at $T = \infty$, The model is expanding ,shearing, rotating and anisotropic in general , Since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{2} \neq 0$. The model does not approach to isotropy for large value T . Also, it is observe that when $n = -2$, then expansion θ tends to zero and the energy density of matter $\Omega_m \longrightarrow \infty$ $\Omega_r \longrightarrow \infty$ $\Omega \longrightarrow \infty$ and for radiation universe, the energy density of matter $\Omega_m \longrightarrow \infty$ $\Omega_r \longrightarrow \infty$ $\Omega \longrightarrow \infty$ and $\Omega_r \longrightarrow 0$ for dust model only. The presence of magnetic field expands the universe, the density increases and pressure decreases.

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