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EXISTENCE AND NONEXISTENCE OF BIFURCATION OF PERIODIC TRAVELLING WAVE SOLUTIONS OF NONLINEAR DISPERSIVE LONG WAVE EQUATION

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Abstract: - This article studied the existence and nonexistence of bifurcation of periodic travelling wave solutions of nonlinear dispersive long wave equation by using Lyapunov-Schmidt reduction. We determined the conditions for the existence of regular solutions for the reduced equation corresponding to the main problem. 2010 Mathematics Subject Classification. 34K18, 93C10.

Keywords: Local bifurcation theory, Local Lyapunov-Schmidt method, nonlinear dispersive long wave equation.

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INTRODUCTION

In [2] Boiti introduced the following system of nonlinear partial differential equations

$u_{ty} + v_{xx} + \frac{1}{2}\,(u^2)_{xy}=0,$

$v_t+(uv + u + u_{xy})_x=0.$ (1.1)

Which describe the nonlinear dispersive long wave in (2+1)-dimension. System (1.1) has interest in fluid dynamics**.**Also, good understanding of the solutions for system (1.1) is very helpful to coastal and civil engineers in applying the nonlinear water model to coastal harbor design.There are many studies of system (1.1) by different ways. In [6] Paquin and Winternitz studied the similarity solutions of system (1.1) by using symmetry algebra and the classical theoretical analysis. In [9] Tang and Lou studied the abundant localized coherent structures of a (2+1)-dimensional dispersive long-wave equation by using the variable separation approach. Chen and Yong in [3] obtained Several families of analytical solutions dispersive long-wave equation,their study based upon the extended projective Riccati equations method. In [5] Fan used the ansatz-based method to obtained some exact solutions of the dispersive long wave equation. Yomba In [10] obtained some new soliton-like solutions of the (2 + 1)-dimensional spaces long wave equation by using the improve extended tanh method. In [7] Rong and Tang studied the bifurcation of solitary and periodic waves for (2+1)-dimension nonlinear dispersive long wave equation by using the bifurcation theory of planar dynamical systems. In this paper we investigate the existence of bifurcation of periodic travelling wave solutions of the (2+1)-dimensional dispersive long-wave equation in some domain of parameters by using the Lyapunov-Schmidt method**.**

Definition 1.1 [8] Suppose that *E* and *F* are Banach spaces and A:$E \to F$ be a continuous linear operator.The operator *A* is called Fredholm,if

1-The kernel of *A*, ker(*A*), is finite dimensional;

2-The Range of *A*, Im(*A*),is closed in *F*;

3-The Cokernel of *A*, $\left(Coker(A) = {}^{F}\!/_{Im(A)}\right)$ is finite dimensional.

Theorem 1.1 [1] suppose X and Y are real Banach spaces and $F(x,\lambda)$ is a C^1 map defined in a neighborhood U of a point (x_0,λ_0) with range in Y such that $F(x_0,\lambda_0) = 0$ and $F_x(x_0,\lambda_0)$ is a linear Fredholm operator. Then all solutions (x,λ) of $F(x,\lambda) = 0$ near (x_0,λ_0) (with λ fixed) are in one-to-one correspondence with the solutions of a finite-dimensional system of N_1 real equations in a finite number N_0 of real variables. Furthermore, N_0=dim(ker L) and N_1=dim(coker L), $(L = F_x(x_0, \lambda_0))$.

Definition 1.2 [8]

If A is Fredholm operator, the index of A is the integer

ind(A)=dim(ker(A)) – Codim(Range(A))

or equivalently

ind(A)=dim(ker(A)) – dim(Coker(A))

Definition 1.3 [8] The Nonlinear operator $F: U \subset X \to Y$ is called Fredholm if the first Fréchet derivative $dF(x)$ is a Fredholm for every $x \in U$. The index of the nonlinear Fredholm operator F is equal to the index of the linear operator $dF(x)$.

To study the bifurcation of periodic travelling wave solutions of system (1.1) we first consider the travelling wave solutions in the form of

$u(x,y,t)=u(\xi)$, $v(x,y,t)=v(\xi)$, $\xi=x+y-ct$

to reduce system (1.1) into following system,

$$-cu'+v''+(uu''+(u')^2)=0,$$

$$-cv'+(uv+u+u'')'=0. \qquad (1.2)$$

Where c denotes the wave speed and $(\,' = \frac{d}{d\xi})$. From the second equation of system (1.2) we have

$$\frac{1}{c}(uv+u+u'')' = v' \qquad (1.3)$$

Substitute (1.3) into first equation of (1.2) we get

$$-cu''+[\frac{1}{C}(uv+u+u'')'\,]'+(uu''+(u')^2)=0 \qquad (1.4)$$

by using the transformation $v = 1 + u + u^2$, equation (1.4) became

$$u''''+\lambda_2 u''+\lambda_3(uu''+(u')^2)+3(u^2u''+2u(u')^2)=0 \qquad (1.5)$$

Where, $\lambda_2 = (2 - c^2)$, $\lambda_3 = (2 + c)$

In our study we assume that *u* and *v* are periodic functions,

$u(\xi) = u(\xi + T)$, $v(\xi) = v(\xi + T)$, $T = 2\pi$.

To study the bifurcation of periodic travelling wave solutions of system (1.2) we shall study the bifurcation solutions of equation (1.5). In the next section we will apply the Lyapunov-Schmidt reduction to reduce equation (1.5) into an equivalent system of two nonlinear algebraic equations.

2. Reduction to the bifurcation equation corresponding to equation (1.5)

To apply Lyapunov-Schmidt method for equation (1.5) we first rewrite equation (1.5) in the operator equation form

$$F(u,\lambda) = u'''' + \lambda_2 u'' + \lambda_3 (uu'' + (u')^2) + 3(u^2u'' + 2u(u')^2) \qquad (2.1)$$

Where $F:\Pi_4([0,2\pi],\mathbb{R}) \rightarrow \Pi_0([0,2\pi],\mathbb{R})$ is a nonlinear Fredholm operator of index zero, $\Pi_4([0,2\pi],\mathbb{R})$ is the space of all periodic continuous functions that have derivative of order at most four, $\Pi_0([0,2\pi],\mathbb{R})$ is the space of all periodic continuous functions, $\mathbb{R}$ is the real space, $u = u(\xi)$, $\xi \epsilon [0,2\pi]$ and $\lambda = \lambda_2$. We note that the bifurcation solutions of equation (1.5) is equivalent to the bifurcation solutions of operator equation [8]

$$F(u,\lambda) = 0 \qquad (2.2)$$

The first step in this reduction is to determine the linearized equation corresponding to the equation (2.2), which is given by the following equation,

$$Lh = 0 \;, \qquad h \in \Pi_4([0,2\pi],\mathbb{R}),$$

$$L = F_u(0,\lambda) = \frac{d^4}{dx^4} + \lambda_2 \frac{d^2}{dx^2},$$

So the point $\lambda_2 = 2$ is a bifurcation point of equation (2,2). Localized parameter λ_2 as follows

$\lambda_2 = 2 + \delta_1$, δ_1 are small parameters

lead to bifurcation along the modes

$$e_1(x) = c_1 \sin(x), \qquad e_2(x) = c_2 \cos(x).$$

Where $\|e_i\|_H$=1 and $c_i=\sqrt{2}$ for i=1,2 and $(H=L_2([0,2\pi],\mathbb{R})$ is a Hilbert space). Let $N=Ker(L)=span\{e_1,e_2\}$, then the space $\Pi_4([0,2\pi],\mathbb{R})$ can be decomposed in direct sum of two subspaces, N and the orthogonal complement to N.

$$\Pi_4([0,2\pi],\mathbb{R})=N\oplus N^{\perp},\quad N^{\perp}=\{\tilde{v}\in\Pi_4([0,2\pi],\mathbb{R}):\tilde{v}\perp N\}.$$

Similarly, the space $\Pi_0([0,2\pi],\mathbb{R})$ can be decomposed in direct sum of two subspaces, N and the orthogonal complement to N.

$$\Pi_0([0,2\pi],\mathbb{R})=N\oplus\tilde{N}^{\perp},\quad \tilde{N}^{\perp}=\{g\in\Pi_0([0,2\pi],\mathbb{R}):g\perp N\}.$$

Accordingly, there exist projections $p:\Pi_4([0,2\pi],\mathbb{R})\to N$ and $I-p:\Pi_4([0,2\pi],\mathbb{R})\to N^{\perp}$ such that $pu=w$, $(I-p)u=\tilde{v}$ and hence every vector $u\in\Pi_4([0,2\pi],\mathbb{R})$ can be written in the form of,

$$u=w+\tilde{v},\quad w=\sum_{i=1}^{2}x_ie_i\in N,\quad \tilde{v}\in N^{\perp},\quad x_i=<u,e_i>_H.$$

(<·,·> is the inner product in Hilbert space $H=L_2([0,2\pi],\mathbb{R})$. Similary, there exists projections $\varrho:\Pi_0([0,2\pi],\mathbb{R})\to N$ and $I-\varrho:\Pi_0([0,2\pi],\mathbb{R})\to\tilde{N}^{\perp}$ such that

$$F(u,\lambda)=\varrho F(u,\lambda)+(I-\varrho)F(u,\lambda),$$

$$\varrho F(u,\lambda)=\sum_{i=1}^{2}v_i(u,\lambda)e_i\in N,\quad (I-\varrho)F(u,\lambda)\in\tilde{N}^{\perp},$$

$$v_i(u,\lambda)=<F(u,\lambda),e_i>_H.$$

Hence equation (2.2) can be written as

$$\varrho F(u,\lambda)=0,$$

$$(I-\varrho)F(u,\lambda)=0.$$

Or

$$\varrho F(w+\tilde{v},\lambda)=0,$$

$$(I-\varrho)F(w+\tilde{v},\lambda)=0.$$

By the implicit function theorem, there exists a smooth map $\theta: N \to N^{\perp}$ such that $\tilde{v} = \theta(w, \lambda)$ and

$$(I - \varrho)F(w + \theta(w, \lambda), \lambda) = 0.$$

So to find the solutions of equation (2.2) in the neighborhood of the point $u = 0$ it is sufficient to find the solutions of the equation,

$$\varrho F(w + \theta(w, \lambda), \lambda) = 0. \qquad (2.3)$$

Equation (2.3) is the bifurcation equation corresponding to equation (1.5).Since,

$$\varrho F(u, \lambda) = \sum_{i=1}^{2} v_i(u, \lambda) e_i = 0, \quad v_i(u, \lambda) =< F(u, \lambda), e_i >_H$$

Then the bifurcation equation can be written in the form of

$$\varrho F(u, \lambda) = \Theta(\xi, \lambda) = \sum_{i=1}^{2} v_i(u, \lambda) e_i = 0, \qquad \xi = (x_1, x_2).$$

Equation (2.1) can be written as

$F(w + \tilde{v}, \lambda)$=$L(w + \tilde{v}) + \mathcal{B}(w + \tilde{v})$

=$Lw + \lambda_3 \, (ww'' + (w')^2) + \; 3(\, w^2 w'' + 2 \, w \, (w')^2) \; + \cdots$

Where

$\mathcal{B}(w + \tilde{v}) = \lambda_3 \, ((w + \tilde{v})(w + \tilde{v})'' + ((w + \tilde{v})')^2) + \; 3(\, (w + \tilde{v})^2 (w + \tilde{v})'' + 2 \, (w + \tilde{v}) \, ((w + \tilde{v})')^2)$

And the dots denote the terms consists the element $\tilde{v}$ and its derivatives. Hence

$$\Theta(\xi, \lambda) = \sum_{i=1}^{2} < Lw + \lambda_3 \, (ww'' + (w')^2) + \; 3(\, w^2 w'' + 2 \, w \, (w')^2) \; , e_i > e_i$$

$$+ \cdots = 0. \qquad (2.4)$$

To find equation (2.4) we expand the summation then substitute $w = \sum_{i=1}^{2} x_i e_i$

In (2.4). After some calculations we find that the bifurcation equation is given by a system of two nonlinear algebraic equations

$-6x_1^3 - 6x_1x_2^2 + q_1\, x_1 = 0,$

$-6x_2^3 - 6x_2x_1^2 + q_2\, x_2 = 0.$ (2.5)

Where, x_i, q_i are real and $Le_i = \sigma_i(\lambda)e_i$, i=1,2.

From theorem (1.1) the solutions of equation (2.2) are in one-to-one correspondence with solutions of the system (2.5).Thus the point

$$\bar{u} = \sum_{i=1}^{2} x_i e_i + \theta\left(\sum_{i=1}^{2} x_i e_i\,, \lambda\right)$$

Is a solution of equation (2.2) if and only if the point (x_1, x_2) is a solution of system (2.5) [8].

3. Analysis of bifurcation of system (2.5)

In this section we will investigate the existence of the regular solutions of system (2.5). We note that system (2.5) has nine solutions consists the origin. It is clear that the two circles

$S^1 = \left\{(x_1, x_2):\quad x_1^2 + x_2^2 = \frac{q_1}{6}\right\}$ and $\hat{S}^1 = \left\{(x_1, x_2):\quad x_1^2 + x_2^2 = \frac{q_2}{6}\right\}$.

Are never intersects for $q_1, q_2 > 0$, this implies that the system (2.5) will be lose four solutions, so the remaining solutions are given below:

$(0, 0)$, $(0, \pm\sqrt{\frac{q_2}{6}})$, $(\pm\sqrt{\frac{q_1}{6}}, 0)$.

These solutions are generate on the surface given by the equation

$216x_1^2x_2^2 + 108x_1^4 + 108x_2^4 - 18x_1^2q_2 - 6x_2^2q_2 - 18q_1x_2^2 - 6q_1x_1^2 + q_1q_2 = 0$.

Moreover, the bifurcation set of system (2.5) is given by the equation

$$q_1q_2(q_1 - q_2) = 0.$$

It follows that the existence of the solutions of system (2.5) depend on the values of the parameters q_1 and q_2 . The change in the values of parameters q_1 and q_2 give rise to the different solutions of system (2.5), so if $q_1 > 0$ and $q_2 > 0$, then system (2.5) has five real solutions,

if $q_1 < 0$ and $q_2 > 0$, then system (2.5) has three real solutions,

if $q_1 > 0$ and $q_2 < 0$, then system (2.5) has three real solutions,

if $q_1 < 0$ and $q_2 < 0$, then system (2.5) has one real solutions.

If $q_1 = q_2$, then system (2.5) has five real solutions when $q_1 > 0$ and one solution if $q_1 < 0$. Accordingly, the linear approximation of the non-zero solutions of equation (1.5) is given in the form of

$$w_{1,2}(x) = \pm\sqrt{\frac{q_1}{3}}\ \sin(x) \quad \text{and} \quad w_{3,4}(x) = \pm\sqrt{\frac{q_2}{3}}\ \cos(x).$$

Where, $w_{1,2}(x)$ refer to the first and second solutions (with positive and negative sign). These solutions have the following geometric representations

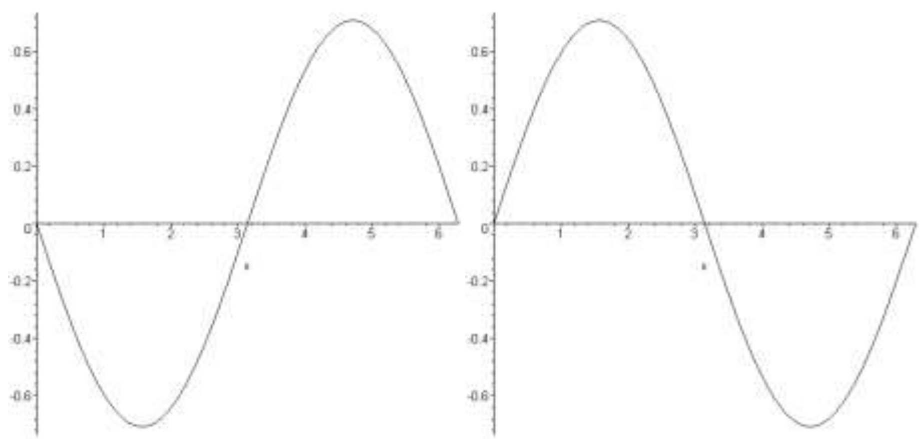

Fig.(1) The graph of the functions $\pm\sqrt{\frac{q_1}{3}}\ \boldsymbol{sin}(\boldsymbol{x})$**.**

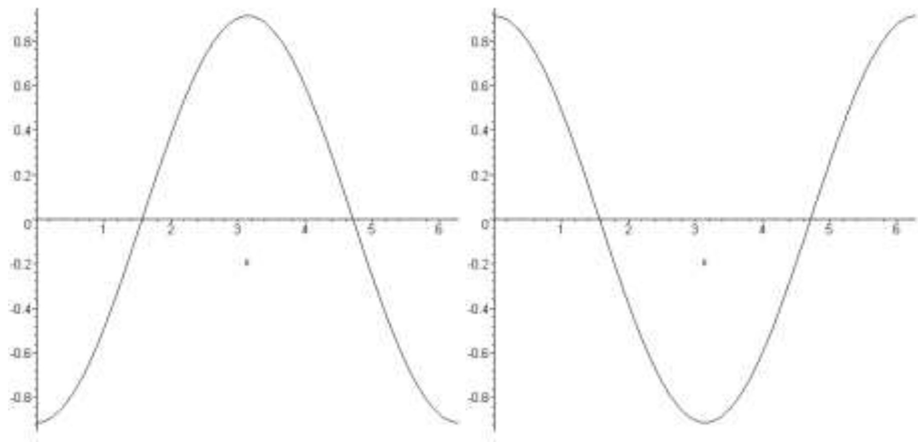

Fig.(2) The graph of the functions $\pm\sqrt{\frac{q_2}{3}}\,cos(x)$.

Since, $v = 1 + u + u^2$ it follows that the function v has the following geometric representations

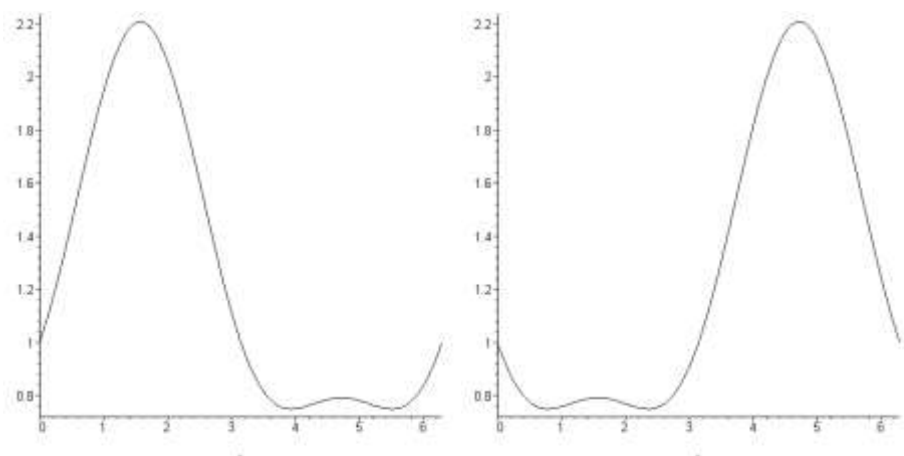

Fig.(3) The graph of the functions *v* when *u*= $\pm\sqrt{\frac{q_1}{3}}\,sin(x)$.

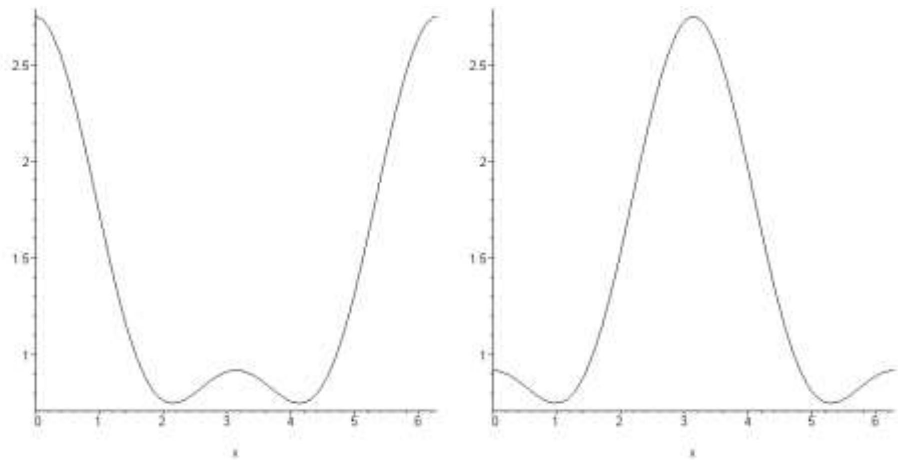

Fig.(4) The graph of the functions *v* when *u*= $\pm\sqrt{\frac{q_2}{3}}\ cos(x)$**.**

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