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A THEORETICAL STUDY OF DIMENSIONAL EFFECTS IN COLLECTIVE EXCITATIONS OF OPTICALLY TRAPPED QUASI-2D BOSE GAS AND EVALUATION OF MANY-BODY EFFECTS TO EQUATION OF STATE AND CHEMICAL POTENTIAL OF THE GAS

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Abstract: - Using the theoretical formalism of **Ying Hu and Z. Liang [arXiv:1103.3079v1[cond.Mat.quan.gas] 16 march 2011]**, we have studied the optically trapped quasi-2D Bose gas and evaluated chemical potential of the gas. Taking the ground state energy and differentiating with respect to total number of particles, chemical potential was evaluated. After proper asymptotic analysis, it was found that in the asymptotic 3D regime, the value is consistent with 3D **Lee-Huang-Yang result (Phys. Rev 106,1135 (1957))**. On the other hand in the opposite pure 2D limit it stands in good agreement with **C. Mora and Y. Castin result [PRL 102, 180404 (2009)]**. The calculation also gives first correction to the 3D mean-field (MF) equation of state arising from the 2D effect. In an another calculation, using theoretical formalism of **Sylvan Nascimbeue etal.(arXiv:1006.4053v1 [cond.mat.quant. gas) 21 June 2010)**, we have evaluated equation of state of a Bose gas in an optical lattice in the Mott-insulator regime. Our evaluated results are in good agreement with other theoretical workers. We have also evaluated the equation of state of the 2D homogeneous Bose gas. Our theoretically evaluated results were compared with EOS(Schivk formulae), perturbative EOS and EOS(A. Boudjimnaa). Our theoretically evaluated results are in good agreement with these workers.

Keywords: Dimensional effect, collective excitations in optically trapped quasi-2D Bose gas, BKT (Berizimskii-Kosterlitz-Thoules) transition ,Cylindrically symmetric harmonic trap, Linerized hydrodynamic equation, Lattice induced dimensional crossover, Anisotropic 3D behavior, Density dependent effective coupling constant, Weak quasi-2D regime, Popov's equation of state (EOS), Mott-insulator regime.

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INTRODUCTION

Recent discovery of quasi-2D ultra cold Bose gas[1] has gave a new light in understanding of low dimensional many-body effects[2]. In this, 2D strongly correlated quantum systems there is interplay between dimensionality and quantum fluctuation. Because of this some remarkable phenomenon has been observed and prominent among them are high-temperature superconductivity[3] and Berezinski-Kosterlitz-Thouless (BKT) transition[4]

Generally in a harmonically trapped 2D gas, tuning interaction can qualitatively change the nature of the transitions. For an interacting gas, the emergence of a topological order below some critical temperature gives rise to super fluidity mediated by the BKT transitions whereas BEC is precluded by the requisite critical density being infinite. On the other hand, an ideal gas cannot exhibit super fluid dynamics, but can undergo BEC resulting in a homogeneous phase. Using a harmonically trapped 2D Bose gas, one identifies the low temperature phase transition over a decade of interaction strength. It can also be shown that the interaction driven BKT transition smoothly converges on to the purely quantum- statistical BEC transition in the limit of vanishing interactions.

It has been observed that dimensionality also played an important role and therefore one is interested to get the spectroscopic diagnostics of this effect. This effect can be seen by evaluating the collective frequencies to equation of state to see how 3D equation of state change due to many-body effects[5]. According to CDKSS (Castini-Dum-Kagan-Surkov-Shlyapnikov) scaling ansatz[6,7], a 2D harmonically trapped Bose gas with gδ^2(**r**) interaction reveals the hidden Pitaevskii-Rosch symmetry[8,9] in an associated classical field theory. The frequency shift in this mode can provide a signature of quantum anomaly emerging upon the quantization when quantum fluctuation significantly modifies the scattering length[10].

MATERIALS AND METHODS

One takes the effective Hamiltonian[11,12]

$$H_o = \Sigma_j [\frac{p^2_{j\perp}}{2m} + \frac{p^2_{jz}}{2m^*}] + \frac{m}{2}\Sigma_j \{\omega^2_\perp (x^2_j + y^2_j) + \omega^2_z z^2_j\} + g\Sigma_{j<k}\delta^3(r_{jk}) \tag{1}$$

where g is the lattice renormalized 3D coupling constant, ω_z is axial frequency, $\omega_\perp$ is radial frequency. 1D optical lattice is along the horizontal axis (z-axis) with a BEC in an elongated harmonic trap. One is interested to study the collective oscillations in the presence of optical lattice. The super fluid hydrodynamic analysis[12] shows that the axial optical potential mark no effect on transverse modes for an elongated trap which has been examined experimentally[13]. m^* is the effective mass[14]. This can be changed if one can go quasi-2D regime. Using many-body theory[5], one can have 2D position dependent correction

$$H_1 = \Sigma_{j<k} g_1(r_j)\delta^3(r_{jk}) \qquad (2)$$

Now, the frequencies of collective excitations can be obtained from the equation of motion

$$\frac{d^2F_B}{dt^2} + (2\omega_\perp)^2 F_B = \frac{4}{m}[H_o - \Sigma_j(\frac{p^2{}_{jz}}{2m^*} + \frac{m}{2}\omega^2{}_z z^2{}_j)$$
$$-\Sigma_{j<k}\{r_j^{\rightarrow}\cdot\nabla g_1(r_j^{\rightarrow}) + [\nabla g_1(r_j^{\rightarrow})]\cdot r_j^{\rightarrow}\}\delta^3(r_{jk}^{\rightarrow})] \qquad (3)$$

Where $F_B = \Sigma_i(x^2{}_i + y^2{}_i)$ (4)

Is the excitation operator of the transverse breathing mode. Equation (3) is equally valid classically. It is a pure 2D system within the classical field description, only H_o survives on the right hand side of equation (3). The mode operator oscillates universally with $2\omega_\perp$ as required by Pitaveskii-Rosch symmetry (PRS)[8,9]. It has been also seen that universal oscillation with $2\omega_\perp$ also persists in a 3D elongated dilute Bose gas described[13] by a Hamiltonian H_o $(\frac{\omega_z}{\omega_\perp} \ll 1)$. The oscillation frequency can be shifted from $2\omega_\perp$ by the emerging correction $g_1(r^{\rightarrow})$.

One calculates the collective oscillations of a quasi-2D Bose gas in an optical lattice given by

$$V_{opt} = sE_R \sin^2(q_B z) \qquad (5)$$

$$V_{ho}(r^{\rightarrow}) = \frac{m}{2}(\omega^2{}_\perp x^2 + \omega^2{}_\perp y^2 + \omega^2{}_z z^2) \qquad (6)$$

$$E_R = \frac{\hbar^2 q^2{}_B}{2m}, q_B = \frac{\pi}{d} \qquad (7)$$

Here, V_{ho} (**r**) is a superimposed cylindrically symmetric harmonic trap. The lattice period is fixed by q_B with d is the lattice constant. S is a dimensionless factor due to intensity of laser beam. E_R is the recoil energy, $\hbar q_B$ being the Bragg momentum.

Now, one can take linear zed hydrodynamic equation for density fluctuation $\delta n(r^{\rightarrow},t)$ to the quasi-2D regime from ref 13

$$m\frac{\partial^2 \delta n}{\partial t^2} - \nabla[n\nabla(\frac{\partial \mu_{Q2D}}{\partial n}\delta n)] = 0 \quad (8)$$

$$\nabla \equiv (\nabla_{\perp}, \nabla_z \sqrt{\frac{m}{m^*}}) \quad (9)$$

Evaluation of chemical potential and equation of state in quasi-2D trapped Bose gas

The 3D density n(**r**) is determined from the ground state value of the chemical potential μ_o fixed by the proper normalization of n(**r**) where μ_o is given by

$$\mu_o = \mu_{Q2D}[n(r^{\rightarrow})] + V_{ho}(r^{\rightarrow}) \quad (10)$$

Now, in order to get the hydrodynamic analysis on collective oscillations one has to know the equation of motion. For this, one has to start with the grand partition function of an optically trapped quasi-2D Bose gas in the absence of harmonic potential which is written as[14,15]

$$Z = \int D[\psi^*, \psi] e^{-S[\psi^*,\psi]/\hbar} \quad (11)$$

$$S[\psi^*,\psi] = \int d\tau \int d^3 r^{\rightarrow} \psi^*(r^{\rightarrow},\tau)[\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m} + V_{opt}(r^{\rightarrow}) + \frac{g_e}{2}[\psi(r^{\rightarrow},\tau]^2]\, \psi(r^{\rightarrow},\tau) \quad (12)$$

Here S is an action functional of $[\psi^*(r^{\rightarrow},\tau), \psi(r^{\rightarrow}\tau)]$ which collectively denote the complex functions of space and imaginary time τ. g_e stands for the two-body coupling constant in an axial optical confinement. Using the path-integral approach[16], one finds within the tight-binding approximation and Bogoliubov theoretical framework[17] the ground state energy E_g

$$\frac{E_g}{V} = \frac{1}{2}\tilde{g}_e n^2[1 + \frac{m\tilde{g}_e}{2\pi^2\hbar^2 d} F(\frac{2t}{\tilde{g}_e n})] \quad (13)$$

With

$$F(x) = \frac{(x+1)}{2}[(3x+1)\arctan(\frac{1}{\sqrt{x}}) - 3\sqrt{x}]$$

$$-\frac{\pi}{2}[\frac{x}{2x+1+2\sqrt{x(x+1)}}] - \pi \arcsin h(\sqrt{x}) + 2\int_0^{\sqrt{x}} \frac{\tan^{-1}(z)}{z} dz \quad (14)$$

Where t denotes the tunneling rate, and n refers to the condensate density. $\tilde{g}_e$ is written as

$$\tilde{g}_e = g_e[d \int_{-d/2}^{d/2} \omega^4(z)dz] = g_e(d/\sqrt{2\pi}\sigma) \quad (15)$$

$$\omega(u) = \exp[-\frac{u^2}{2\sigma^2}] / \pi^{\frac{1}{4}} \sigma^{\frac{1}{2}} \quad (16a)$$

$$\frac{d}{\sigma} = \pi s^{\frac{1}{4}} \exp(-1/4\sqrt{s}) \quad (16b)$$

Here $\omega(u)$ is a variational Gauassian ansataz, $\frac{d}{\sigma}$ minimizes the free energy functional with respect to σ. The ground state energy in equation (13) experiences a lattice induced dimensional crossover governed by the parameter $(\frac{2t}{\tilde{g}_e n})$. In the limit $(\frac{2t}{\tilde{g}_e n})$ >>1, the system exhibits anisotropic 3D behavior. On the other hand if $(\frac{2t}{\tilde{g}_e n})$ <<1, it shows quasi-2D regime. Now from 3D to 2D transition, the complete behavior is manifested as 2D character which gives rise to effective coupling constant[18]

$$g_e = \frac{2\sqrt{2\pi}\hbar^2 d}{m} \frac{1}{a_{2D}/a_{3D} + (1/\sqrt{2\pi})\ln[1/n_{2D}a^2_{2D}]} \quad (17)$$

Where n_{2D} =nd is the surface density and $a_{2D} = \sigma$ is the effective 2D scattering length. The logarithmic density-dependence term in equation (17) is 2D many-body effects. Its importance is governed by the ratio a_{2D}/a_{3D} which controls the dimensional cross-over in hard-core interactions[5]. For a_{2D}/a_{3D} >>1, one has pure 3D collision and density independent coupling constant is given by

$$\tilde{g} = \frac{d}{\sqrt{2\pi\sigma}} \frac{4\pi\hbar^2 a_{3D}}{m} \quad 18(a)$$

$$g_e = g_{2D} d \quad \text{with} \quad g_{2D} = \frac{\frac{4\pi\hbar^2}{m}}{\ln(1/a^2{}_{2D} n_{2D})} \quad 18(b)$$

Linearly expanding equation (17) with respect to a_{2D}/a_{3D}, we have

$$\tilde{g} = \tilde{g}[1 - \frac{1}{\sqrt{2\pi}} \frac{a_{3D}}{a_{2D}} \ln(\frac{1}{n_{2D} a^2{}_{2D}})] \quad 18(c)$$

This shows the logarithmic dependence on the gas parameter which constitutes the leading 2D correction to 3D coupling constant $\tilde{g}$. Now μ_{Q2D} can be calculated by the expression

$$\mu = \frac{\partial E_g}{\partial N} \quad 19(a)$$

From equation (13) and taking the asymptotic 3D regime where $\frac{2t}{\tilde{g}n}$ >>1 and a_{3D}/a_{2D}>>1 one gets

$$\mu = \tilde{g}n[1 + (32m^* / 3\sqrt{\pi}m)\sqrt{a^2{}_{3D} n}] \quad 19(b)$$

This is consistent with 3D Lee-Huang-Yang (LHY) result[18] where $\frac{2t}{\tilde{g}n}$ <<1 and a_{3D}/a_{2D}<<1, one gets

$$\mu = \frac{4\pi\hbar^2 n_{2D} / m}{\ln n_{2D} a^2{}_{2D}}[1 - \frac{\ln(\ln(1/n_{2D} a^2{}_{2D}) - B}{\ln(1/n_{2D} a^2{}_{2D})} - \frac{\ln(\ln(1/n_{2D} a^2{}_{2D}) - B}{(\ln^2(1/n_{2D} a^2{}_{2D})}] \quad 19(c)$$

Where $$B = 1 - \ln(mt / n_{2D} 2\pi\hbar^2) \quad 19(d)$$

For quasi -2D regime, we have

$$\mu_{Q2D} = \tilde{g}n[1+\frac{1}{\sqrt{2\pi}}\frac{a_{3D}}{a_{2D}}(\frac{1}{2}-\ln(\frac{1}{n_{2D}a^2{}_{2D}}))] \quad (20)$$

Equation of state and chemical potential for weakly interacting Bose gas:

The equation of state of a weakly interacting Bose gas can be expressed in the grand canonical ensemble[19,20]

$$P(\mu,T) = \frac{K_B T}{\lambda^3{}_{dB}(T)} g(\varsigma) \quad 21(a)$$

$$\varsigma = e^{\frac{-\mu}{k_B T}} \quad 21(b)$$

$$\lambda_{dB}(T) = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$$ (thermal de'Brogle wavelength) 21(c)

One obtains the global chemical potential value $\mu^0 = 0.10 k_B T$ by fitting the ^{7}Li profile in the non-condensed region $|z| > 50\ \mu m$ with Bose function given by

$$P(\mu_z,T) = \frac{k_B T}{\lambda^3{}_{dB}(T)} g_{\frac{5}{2}}(\varsigma_z) \quad 21(d)$$

$$\varsigma_z = e\frac{-\mu^0}{k_B T}\exp(\frac{m\omega^2{}_z z^2}{2k_B T}) \quad 21(e)$$

$$g_{\frac{5}{2}}(z) = \sum_{k=1}^{\infty}\frac{z^{-k}}{k^{\frac{5}{2}}} \quad 21(f)$$

Equation of state of the Bose-Hubbard model in the Mott-insulator regime:

In this regime the equation of state is determined with different atom numbers N_a =1.0x10^5, N_b=2.0x10^5 and N_c=3.5x10^5 having three images labeled a,b and c. The chemical potential differences between different images are[21-23]

$$\mu^0{}_b - \mu^0{}_a = 0.56U \quad 22(a)$$

$$\mu^0{}_c - \mu^0{}_b = 0.61U \quad 22(b)$$

$$\mu^0{}_a = 1.51U \quad (22(c))$$

U is the on-site interaction. The chemical potential of a weakly-interacting Bose-Einstein condensate is given by

$$\mu = \frac{4\pi\hbar^2 a_{77} n}{m_7} \quad 22(d)$$

Where m_7 is the mass of ^7Li atom and a_{77} is the scattering length describing s-wave interactions between ^7Li atoms. N is number of atoms in the cloud. Results are shown in table T1 and table T2.

Equation of state of the 2D homogenous Bose gas:

In 2D Bose gas, the interaction parameter (g_d =g_2) depend logarithmically on the chemical potential as[24,25]

$$g_2 = [\frac{4\pi\hbar^2}{m} \frac{1}{\log(\frac{2\hbar^2}{m\mu a^2})}] \quad (23)$$

Where a is 2D scattering length among the particles and g_2 is two- body T-matrix. The chemical potential μ is given by

$$\mu = \mu(0)[1 - \frac{3}{4\pi n}(\frac{mc_s}{\hbar})^2] \quad (23a)$$

N=total density

c_s =sound velocity

Under the weak interaction equation (23a) reduces to well known Popav's equation of state (EOS)[26]

$$n \approx \frac{m\mu}{4\pi\hbar^2} = \log(\frac{4\hbar^2}{m\mu a^2 e^{2\gamma+1}}), \gamma = Euler's \text{ constant} \quad 23(b)$$

The ground state energy is given by

$$E(n)/N = \frac{2\pi\hbar^2 n}{m}\{\log(na^2) - \frac{1}{2} - \frac{3}{4\pi n}(\frac{mc_s}{\hbar})^2[-1+\log(na^2)] \qquad 23(c)$$

Now, the value of (na^2) is computed as a function of $\frac{\mu m a^2}{\hbar^2}$ and the results are shown in the table T3.

Results and Discussion

In this paper, using the theoretical formalism of Ying Hu and Z. Liang[27], we have theoretically studied the optically trapped quasi-2D Bose gas. In this formalism, we have studied the chemical potential μ_{Q2D}. μ_{Q2D} has been determined by differentiating the ground state energy with respect to total number of atoms. The analytical solution is consistent with 3D Lee-Huang-Yang (LHY) result[18]. It is also observed that in the opposite pure 2D limit, the chemical potential of 2D Bose gas is in the good agreement with the result of C. Mora and Y Castin[28]. The results also give the first order correction to the 3D- mean-field (MF) equation of state arising from the 2D effect. Using the theoretical formalism of Sylvain Nascimbene etal.[29], we have studied the equation of state of weakly –interacting Bose-gas and calculated the global chemical potential μ^0 by taking the experimental data of ^{7}Li Bose gas[30]. We have also calculated equation of state in an optical lattice in the Mott-insulator region by computing $\frac{P}{(U(\lambda/2)^{-3})}$ as a function of $\frac{\mu}{U}$. The results are shown in **table T1 and T2**. In an anther calculation, we have computed the equation of state of 2D homogeneous Bose-gas and compared our results with that of equation of state (EOS) of SchicK[31], Perturbative EOS[32] and EOS obtained by A Boudjemaa etal.[33]. The results are shown in **table T3.** Our theoretically obtained results are in good agreement with these workers. There is some recent results[34-40] which also reveals the similar behavior.

CONCLUSION From the above theoretical investigations and analysis, we have come across the following conclusions;

(1) We have studied the chemical potential of quasi-2D optically trapped Bose gas and found that under asymptotic 3D regime the chemical potential is consistent with 3D Lee-Huang-Yang (LHY) result.

(2) We also observed that under opposite pure 2D limit, the chemical potential of 2D Bose gas reduces to a well known value of C. Mora and Y Castin.

(3)We have also noticed that applying similar scheme to the weak-quasi 2D regime it gives first correction to the 3D mean field (MF) equation of state arising from the 2D effect

(4)We have also determined the global chemical potential μ^0 and equaton of state (EOS) of Bose gas in optical lattice and Mott-insulator regime. Here we have shown the term $\frac{P}{(U(\lambda/2)^{-3})}$ as a function of $\frac{\mu}{U}$. λ is laser length. In the Mott-insulating regime there is abrupt increase of $\frac{P}{(U(\lambda/2)^{-3})}$ values.

(5)We have also calculated equation of state (EOS) of 2D homogeneous Bose gas by computing the value of (na^2) as a function of $\frac{\mu n a^2}{\hbar^2}$. On comparing our results with those of Schick's, perturbative EOS and EOS of A. Boudjemaa etal. Our results are in good agreement with these workers,

(6)Our theoretical analysis show that 2D many-body effects can be visible in the calculation of chemical potential which indicates that optically trapped Bose gas gives transition from 3D to quasi-2D regime. This type of results observes dimensional effect. The results also outline an interplay between dimensionality and quantum fluctuation in low-dimensional quantum correlated system like BEC.

Table T1: Determination of the global chemical potential difference $(\mu^0{}_c - \mu^0{}_b)$ by superposing the equation of states $P/(U(\lambda/2)^{-3})$

$(\mu^0{}_c - \mu^0{}_b)$ /U	$P/(U(\lambda/2)^{-3})$
-4.0	0.078
-3.0	0.094
-2.0	0.154
-1.0	0.462
-0.05	0.989
0.00	1.586

0.05	2.643
1.0	3.559
2.0	4.286
3.0	5.159

Table T2: Determination of equation of state (EOS) of a Bose gas in a optical lattice in the Mott-insulator regime

$\frac{\mu}{U}$	$\frac{P}{(U(\lambda/2)^{-3})}$
-4.00	0.0328
-3.00	0.0463
-2.00	0.0589
-1.00	0.0886
-0.05	0.0974
0.00	1.032
1.00	1.256
2.00	1.486
3.00	2.293
4.00	3.495
5.00	4.328

Table T3: Determination of equation of state (EOS) of 2D homogeneous Bose gas

$\frac{\mu n a^2}{\hbar^2}$	**(na^2)**			
	Schick EOS	Perturbative EOS	Boudjemaa EOS	Our results
0.00	0.002	0.003	0.004	0.006
0.01	0.008	0.009	0.010	0.016
0.02	0.021	0.015	0.018	0.024
0.025	0.026	0.022	0.025	0.031
0.03	0.033	0.028	0.029	0.036
0.035	0.037	0.032	0.032	0.041
0.04	0.042	0.038	0.036	0.047
0.045	0.045	0.040	0.043	0.052
0.05	0.048	0.045	0.048	0.057
0.055	0.052	0.047	0.050	0.062
0.06	0.054	0.050	0.052	0.067
0.065	0.056	0.052	0.055	0.070
0.07	0.059	0.055	0.059	0.072

0.075	0.062	0.058	0.061	0.075
0.080	0.065	0.062	0.063	0.077
0.085	0.070	0.065	0.067	0.079
0.090	0.076	0.067	0.070	0.085

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