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AN EVALUATION OF FREQUENCY SHIFT OF COLLECTIVE EXCITATIONS OF OPTICALLY TRAPPED QUASI-2D BOSE GAS AND EVALUATION OF TEMPERATURE DEPENDENT FREQUENCY AND DAMPING RATE OF CONDENSATE BREATHING MODE.

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Abstract: - Using the theoretical formalism of **Ying Hu and Z. Liang [arXiv:1103.3079v1[cond.Mat.quan.gas] 16 march 2011]**, we have theoretically evaluated the frequency shift of optically trapped quasi-2D Bose gas. The frequency shifts were calculated in three different ways: (i) Taking the transverse breathing mode in an elongated trap using TF approximation. (ii) Taking the lowest compression mode in disk like geometry using TF approximation. (iii) Taking the amplitude of oscillating frequency with non-linearity effect. We observed that from (i) and (ii) ways the results of frequency shifts are identical. In the third evaluation, the frequency shift increases very sharply with A (the amplitude of oscillating frequency of the cloud. Our theoretically evaluated results are in good agreement with other theoretical workers. Using the time dependent Gross-Pitaevskii (GP) equation, we have determined the frequency of the rubidium atoms as a function of atoms N for even parity m=0 and m=2.Our theoretically evaluated values are in good agreement with the experimental data. In other calculation, we have determined the mode frequency as a function of asymmetry parameter λ. Our calculation shows that there are two types of mode frequency ω_+ and ω_-. ω_- Increases very sharply and attains a constant values whereas ω_+ increases very slowly but enhances very sharply. Our evaluation of temperature dependent frequency and damping rate of condensate breathing mode are also in good agreement with the experimental data.

Keywords: Frequency shift, Gross-Piteavskii equation, Transverse breathing mode, Lowest compressional mode, Dimensional effect, collective excitations in optically trapped quasi-2D Bose gas, BKT (Berizimskii-Kosterlitz-Thoules) transition ,Cylindrically symmetric harmonic trap, Linerized hydrodynamic equation, Lattice induced dimensional crossover, Anisotropic 3D behaviour, Density dependent effective coupling constant, Weak quasi-2D regime, Popov's equation of state (EOS), Mott-insulator regime.

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INTRODUCTION

In an earlier paper[1], we have studied collective excitations of optically trapped quasi-2D Bose gas and determined the equation of state and chemical potential of the gas. We have also determined the equation of state of weakly-interacting Bose gas and calculated the global chemical potential μ^0 by taking the experimental data of ^{7}Li Bose gas[2]. In another calculation, we have evaluated equation of state of optically trapped 2D Bose gas in Mott-insulator region. Our theoretically evaluated results are in good agreement with other theoretical workers. In the case of homogeneous 2D Bose gas, our theoretically evaluated equation of state (EOS) match with the EOS of Schick[3], perturbative EOS[4] and EOS obtained by A. Boudjemaa etal[5].

In this paper, using the theoretical formulation of Ying Hu and Z. Liang[6], we have evaluated the frequency shift of collective excitation of quasi-2D optically trapped Bose gas. The shift has been calculated through the solution of linearized hydrodynamic equation.[7-9] The frequency shift has been calculated in three different ways: (i) Taking the transverse breathing mode in a very elongated trap using Thomas-Fermi (TF) approximation[10].(ii) Taking lowest compression mode in a disk like geometry and TF radius is along Axial direction. (iii) Using the amplitude of oscillation and taking the effect of non-linearity.

MATERIALS AND METHODS

One takes the effective Hamiltonian[11,12]

$$H_o = \Sigma_j [\frac{p^2{}_{j\perp}}{2m} + \frac{p^2{}_{jz}}{2m^*}] + \frac{m}{2}\Sigma_j \{\omega^2{}_\perp (x^2{}_j + y^2{}_j) + \omega^2{}_z z^2{}_j\} + g\Sigma_{j<k}\delta^3(r_{jk}) \tag{1}$$

where g is the lattice renormalized 3D coupling constant, ω_z is axial frequency, $\omega_\perp$ is radial frequency. 1D optical lattice is along the horizontal axis (z-axis) with a BEC in an elongated harmonic trap. One is interested to study the collective oscillations in the presence of optical lattice. The super fluid hydrodynamic analysis[12] shows that the axial optical potential makes no effect on transverse modes for an elongated trap which has been examined experimentally[13]. m^* is the effective mass[14]. This can be changed if one can go quasi-2D regime. Using many-body theory[5], one can have 2D position dependent correction

$$H_1 = \Sigma_{j<k}\, g_1(r_j)\delta^3(r_{jk}) \tag{2}$$

Now, the frequencies of collective excitations can be obtained from the equation of motion

$$\frac{d^2F_B}{dt^2}+(2\omega_\perp)^2F_B=\frac{4}{m}[H_o-\Sigma_j(\frac{p^2{}_{jz}}{2m^*}+\frac{m}{2}\omega^2{}_z z^2{}_j)$$
$$-\Sigma_{j<k}\{r_j^{\to}\cdot\nabla g_1(r_j^{\to})+[\nabla g_1(r_j^{\to})]\cdot r_j^{\to}\}\delta^3(r_{jk}^{\to})] \quad (3)$$

Where $F_B=\Sigma_i(x^2{}_i+y^2{}_i)$ (4)

Is the excitation operator of the transverse breathing mode. Equation (3) is equally valid classically. It is a pure 2D system within the classical field description, only H_o survives on the right hand side of equation (3). The mode operator oscillates universally with 2 $\omega_\perp$ as required by Pitaveskii-Rosch symmetry (PRS)[15,16]. It has been also seen that universal oscillation with $2\omega_\perp$ also persists in a 3D elongated dilute Bose gas described[13] by a Hamiltonian H_o $(\frac{\omega_z}{\omega_\perp}\ll 1)$.

The oscillation frequency can be shifted from $2\omega_\perp$ by the emerging correction $g_1(r^{\to})$.

One calculates the collective oscillations of a quasi-2D Bose gas in an optical lattice given by

$$V_{opt}=sE_R\sin^2(q_Bz) \quad (5)$$

$$V_{ho}(r^{\to})=\frac{m}{2}(\omega^2{}_\perp x^2+\omega^2{}_\perp y^2+\omega^2{}_z z^2) \quad (6)$$

$$E_R=\frac{\hbar^2q^2{}_B}{2m},q_B=\frac{\pi}{d} \quad (7)$$

Here, V_{ho} (**r**) is a superimposed cylindrically symmetric harmonic trap. The lattice period is fixed by q_B with d is the lattice constant. S is a dimensionless factor due to intensity of laser beam. E_R is the recoil energy, $\hbar q_B$ being the Bragg momentum.

Now, one can take linear zed hydrodynamic equation for density fluctuation $\delta n(r^{\to},t)$ to the quasi-2D regime from ref 13

$$m\frac{\partial^2\delta n}{\partial t^2}-\nabla\cdot[n\nabla(\frac{\partial\mu_{Q2D}}{\partial n}\delta n)]=0 \quad (8)$$

$$\nabla \equiv (\nabla_{\perp}, \nabla_z \sqrt{\frac{m}{m^*}}) \quad (9)$$

The 3D density n(**r**) is determined from the ground state value of the chemical potential μ_o fixed by the proper normalization of n(**r**) where μ_o is given by

$$\mu_o = \mu_{Q2D}[n(r^{\rightarrow})] + V_{ho}(r^{\rightarrow}) \quad (10)$$

Now, in order to get the hydrodynamic analysis on collective oscillations one has to know the equation of motion. For this, one has to start with the grand partition function of an optically trapped quasi-2D Bose gas in the absence of harmonic potential which is written as[17]

$$Z = \int D[\psi^*, \psi] e^{-S[\psi^*,\psi]/\hbar} \quad (11)$$

$$S[\psi^*, \psi] = \int d\tau \int d^3 r^{\rightarrow} \psi^*(r^{\rightarrow}, \tau)[\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2 \nabla^2}{2m} + V_{opt}(r^{\rightarrow}) + \frac{g_e}{2}[\psi(r^{\rightarrow}, \tau]^2] \; \psi(r^{\rightarrow}, \tau) \quad (12)$$

Here S is an action functional of $[\psi^*(r^{\rightarrow}, \tau), \psi(r^{\rightarrow}\tau)]$ which collectively denote the complex functions of space and imaginary time τ . g_e stands for the two-body coupling constant in an axial optical confinement. Using the path-integral approach[18], one finds within the tight-binding approximation and Bogoliubov theoretical framework[17] the ground state energy E_g

$$\frac{E_g}{V} = \frac{1}{2} \tilde{g}_e n^2 [1 + \frac{m \tilde{g}_e}{2\pi^2 \hbar^2 d} F(\frac{2t}{\tilde{g}_e n})] \quad (13)$$

With

$$F(x) = \frac{(x+1)}{2}[(3x+1)\arctan(\frac{1}{\sqrt{x}}) - 3\sqrt{x}]$$

$$-\frac{\pi}{2}[\frac{x}{2x+1+2\sqrt{x(x+1)}}] - \pi \operatorname{arcsin} h(\sqrt{x}) + 2\int_0^{\sqrt{x}} \frac{\tan^{-1}(z)}{z} dz \quad (14)$$

Where t denotes the tunneling rate, and n refers to the condensate density. $\tilde{g}_e$ is written as

$$\tilde{g}_e = g_e [d \int_{-d/2}^{d/2} \omega^4(z) dz] = g_e (d / \sqrt{2\pi}\sigma) \quad (15)$$

$$\omega(u) = \exp[-\frac{u^2}{2\sigma^2}] / \pi^{\frac{1}{4}} \sigma^{\frac{1}{2}} \quad (16a)$$

$$\frac{d}{\sigma} = \pi s^{\frac{1}{4}} \exp(-1/4\sqrt{s}) \quad (16b)$$

Here $\omega(u)$ is a variational Gauassian ansataz, $\frac{d}{\sigma}$ minimizes the free energy functional with respect to σ. The ground state energy in equation (13) experiences a lattice induced dimensional crossover governed by the parameter $(\frac{2t}{\tilde{g}_e n})$. In the limit $(\frac{2t}{\tilde{g}_e n})$ >>1, the system exhibits anisotropic 3D behavior. On the other hand if $(\frac{2t}{\tilde{g}_e n})$ <<1, it shows quasi-2D regime. Now from 3D to 2D transition, the complete behavior is manifested as 2D character which gives rise to effective coupling constant[19]

$$g_e = \frac{2\sqrt{2\pi}\hbar^2 d}{m} \frac{1}{a_{2D}/a_{3D} + (1/\sqrt{2\pi}) \ln[1/n_{2D}a^2{}_{2D}]} \quad (17)$$

Where n_{2D} =nd is the surface density and $a_{2D} = \sigma$ is the effective 2D scattering length. The logarithmic density-dependence term in equation (17) is 2D many-body effects. Its importance is governed by the ratio a_{2D}/a_{3D} which controls the dimensional cross-over in hard-core interactions[5]. For a_{2D}/a_{3D} >>1, one has pure 3D collision and density independent coupling constant is given by

$$\tilde{g} = \frac{d}{\sqrt{2\pi\sigma}} \frac{4\pi\hbar^2 a_{3D}}{m} \quad 18(a)$$

$$g_e = g_{2D} d \text{ with } g_{2D} = \frac{\frac{4\pi\hbar^2}{m}}{\ln(1/a^2{}_{2D} n_{2D})} \quad 18(b)$$

Linearly expanding equation (17) with respect to a_{2D}/a_{3D}, we have

$$\tilde{g} = \tilde{g}[1 - \frac{1}{\sqrt{2\pi}} \frac{a_{3D}}{a_{2D}} \ln(\frac{1}{n_{2D} a^2{}_{2D}})] \quad 18(c)$$

This shows the logarithmic dependence on the gas parameter which constitutes the leading 2D correction to 3D coupling constant

Determination of frequency shift of collective excitation of quasi-2D Bose gas:

(i) Taking transverse breathing mode in an elongated trap in TF approximation:

One starts with equation of state for the 3D ground state density which is written as

$$n(r) = n_{TF} - \frac{1}{\sqrt{2\pi}}\frac{a_{3D}}{a_{2D}}[1+\ln(\frac{1}{dn_{TF}a^2{}_{2D}})]n_{TF} \qquad (19a)$$

Where n_{TF} is the Thomas-Fermi density given by

$$n_{TF} = \frac{(\mu_0 - V_{ext}(r))}{g^{\square}} \qquad 19(b)$$

Where μ^0 is the ground state value of the chemical potential fixed by the proper normalization of n(r)

$$V_{ho} = \frac{m}{2}[\omega^2{}_\perp x^2 + \omega^2{}_\perp y^2 + \omega^2{}_z z^2] \qquad 19(c)$$

Here $\omega_\perp$ is radial and ω_z is axial frequency. $g^{\square}$ is lattice renormalized 3D coupling constant. μ_0 is given by equation (10). μ_{Q2D} is chemical potential of quasi-2D Bose gas given by

$$\mu_{Q2D} = g^{\square} n[1+\frac{1}{\sqrt{2\pi}}\frac{a_{3D}}{a_{2D}}(\frac{1}{2}-\ln(\frac{1}{n_{2D}a^2{}_{2D}}))] \qquad 19(d)$$

$$\mu_{Q2D} = g^{\square} n[1+k_{2D}(n)] \qquad 19(e)$$

$$k_{2D}(n) = \frac{a_{3D}}{\sqrt{2\pi}a_{2D}}[((\frac{1}{2}-\ln(\frac{1}{n_{2D}a^2{}_{2D}}))] \qquad 19(f)$$

Now substituting equation 19(f) and 19(a) in equation (8) and retaining only the term linear in $k_{2D}(n)$, we get

$$m\omega^2\delta n + \tilde{\nabla}\square(\tilde{g}n_{TF}\tilde{\nabla}\delta n) = -\tilde{\nabla}^2(\tilde{g}n^2{}_{TF}\frac{\partial k_{2D}}{\partial n_{TF}}\delta n) \qquad (20)$$

Equation (20) is the familiar 3D hydrodynamic equation in the presence of 1D optical lattice. The rhs of equation (20) present a perturbative term. The resulting fractional frequency shift is given by

$$\frac{\delta\omega}{\omega}=\frac{\tilde{g}}{2m\omega^2}[\frac{\int d^3r^{\leftrightarrow}\tilde{\nabla}^2\delta n(n^2{}_{TF}\frac{\partial k_{2D}}{\partial n_{TF}}\delta n)}{\int d^3r^{\leftrightarrow}\delta n^*\delta n} \quad 20(b)$$

Here the integral extend in the region where n_{TF} is positive[20]. Equation also gives the dependence of $\frac{\delta\omega}{\omega}$ on the derivative of $\frac{\partial k_{2D}}{\partial n}$. This also indicates that the leading order correction arises from 2D effects to 3D MF collective frequency. This shows no logarithmic density dependence. According to equation (20) the surface modes are given by

$$\tilde{\nabla}^2\delta n=0 \quad 20(c)$$

This is not perturbed by the 2D effect in the vicinity of 3D regime. Now the transverse breathing mode in an elongated trap is given by

$$\sqrt{\frac{m*}{m}\frac{\omega_z}{\omega_\perp}} \ll 1 \quad 20(d)$$

Now $$\delta n(r^{\rightarrow}) \propto r^2{}_\perp - R^2{}_{TF}/2 \quad 20(e)$$

$$R_{TF}=\sqrt{\frac{2\mu_0}{m\omega^2{}_\perp}} \quad 20(f)$$

R_{TF} is transverse TF-radius, one gets $\omega = 2\omega_\perp$

Putting all these values in equation (20), one gets

$$\frac{\delta\omega}{\omega}=\frac{1}{4\sqrt{2\pi}}\frac{a_{3D}}{a_{2D}} \quad (21)$$

The results are shown in table T1.

(ii)Evaluation of frequency shift using lowest compression mode in disk like geometry using TF approximation:

Now, one looks at the lowest compression mode in disk like geometry given as

$$\sqrt{\frac{m}{m*}}\frac{\omega_z}{\omega_\perp} \gg 1 \qquad 22(a)$$

This is along the axial direction with the zeroth order dispersion, which is written as

$$\omega = \sqrt{\frac{m}{m*}}\sqrt{3}\omega_z \qquad 22(b)$$

In this case, density oscillation will be of the form

$$\delta n(r^{\rightarrow}) \propto z^2 - \frac{Z^2_{TF}}{3} \qquad 22(c)$$

Where

$$Z_{TF} = \sqrt{\frac{2m^*\mu_0}{m\omega^2_z}} \qquad 22(d)$$

Z_{TF} is the TF radius along the axial direction. Finally one obtains

$$\frac{\delta\omega}{\omega} = \frac{1}{6\sqrt{2\pi}}\frac{m^*}{m}\frac{a_{3D}}{a_{2D}} \qquad 22(e)$$

If one puts $\frac{m^*}{m}$ =1.5 and $\frac{a_{3D}}{a_{2D}} \approx$ 1, then, one obtains identical results as shown in Table T1.

(iii) Taking amplitude of oscillation and effect of non-linearity in the collective frequency:

In this case, one writes

$$\frac{\delta\omega}{\omega} = A\delta^2 \qquad 23(a)$$

where A is fractional oscillating amplitude of the harmonically confined atomic cloud. δ is given by

$$\delta = \frac{5}{2}\lambda^2 \frac{(q_- - 2)(q_+ - 4)(q_- - 5)}{(4q_+ - q_-)(q_- - q_+)^2}[-1 + \frac{15}{4}\frac{\lambda^2}{q^2_+}] - \frac{15}{16}\frac{1}{(q_- - q_+)^2}$$
$$[-q_+ + 2\lambda^2 q_+ - 9\lambda^2 + 8]^2 - \frac{9}{4}\frac{(q_- - 4)}{q_+(q_+ - q_-)} - \frac{3}{20}\frac{(q_+ - 3)}{(q_+ - q_-)}[-10\lambda^2 q_+ + 37\lambda^2 + 11q_+ - 54] \quad 23(b)$$

$$q_\pm = 2 + \frac{3}{2}\lambda^2 \mp \frac{1}{2}[9\lambda^4 - 16\lambda^2 + 16]^{\frac{1}{2}} \quad 23(c)$$

From these equations, we calculated the values of q_+ , q_- and δ and the values are given below

$q_+ = 3.24$, $q_- = 24.77$ and $\delta = 3.10$. We computed the values of $\frac{\delta\omega}{\omega}$ as a function of δ and the results are shown in Table T2. We have taken the values of A from paper of Stamper-Kurn and Ketterle[21].

<u>Evaluation of the breathing mode from time dependent Gross-Pitaevskii equation:</u>

We have GP equation[22,23]

$$i\hbar\frac{\partial}{\partial t}\phi(r^{\rightarrow},t) = [\frac{-\hbar^2\nabla^2}{2m} + V_{ext}(r^{\rightarrow}) + g\left|\phi(r^{\rightarrow},t)\right|^2]\phi(r^{\rightarrow},t) \quad 24(a)$$

Its solution is in the form

$$\phi(r^{\rightarrow},t) = e^{-i\mu t/\hbar}[\phi(r^{\rightarrow}) + u(r^{\rightarrow})e^{-i\omega i} + v^*(r^{\rightarrow})e^{i\omega t}] \quad 24(b)$$

Here u and v are complex numbers. Substituting 24(b) in 24(a), one gets two coupled equations

$$\hbar\omega u(r^{\rightarrow}) = [H_0 - \mu + 2g\phi^2(r^{\rightarrow})]u(r^{\rightarrow}) + g\phi^2(r^{\rightarrow})v(r^{\rightarrow}) \quad 24(c)$$

$$-\hbar\omega v(r^{\rightarrow}) = [H_0 - \mu + 2g\phi^2(r^{\rightarrow})]v(r^{\rightarrow}) + g\phi^2(r^{\rightarrow})u(r^{\rightarrow}) \quad 24(d)$$

where $H_0 = [\frac{-\hbar^2\nabla^2}{2m} + V_{ext}(r^{\rightarrow})]$. These coupled equations are used to calculate excitation of the breathing mode. For spherical traps, the solution of equations 24(c) and 24(d) are characterized by quantum numbers n_r, l and m. n_r is the number of radial modes, l is the angular momentum of the excitation and m is its z-component. The lowest solutions of even parity with m=0 and m=2 are obtained for Rubidium atom confined in an axially symmetric trap $(\omega_x = \omega_y = \omega_\perp)$. The asymmetry parameter of the trap is given by $\lambda = \frac{\omega_z}{\omega_\perp}$. The results of frequency in the unit of $\omega_\perp$ of the lowest collective modes of even parity m=0 and m=2 as a function of number of atoms N are shown in table T3.

Evaluation of breathing mode frequency as a function of anisotropic parameter λ for anisotropic trap:

One considers a harmonic and anisotropic trap but with an axis of symmetry along z-axis. One writes potential in the form[24,25]

$$V(x,y,z) = \frac{1}{2}m\omega^2{}_0\rho^2 + \frac{1}{2}m\omega^2{}_z z^2 = \frac{1}{2}m\omega^2{}_0(\rho^2 + \lambda^2 z^2) \quad (25)$$

where $\rho = (x^2 + y^2)$. The anisotropy parameter is given by $\lambda = \frac{\omega_z}{\omega_0}$.

For spherical trap λ=1 and for TOP[26] $\lambda = \sqrt{8}$. For such a trap the equilibrium density is given in the T-F approximation

$$n = \frac{\mu}{v_0}[1 - \frac{\rho^2}{R^2} - \frac{\lambda^2 z^2}{R^2}] \quad 25(b)$$

Here terms have usual meaning. R is the radius of the cloud

$$R^2 = \frac{2\mu}{m\omega^2{}_0} \quad 25(c)$$

We have equation for mode function as

$$\omega^2 \delta n = \omega^2{}_0(\rho\frac{\partial}{\partial\rho} + \lambda^2 z\frac{\partial}{\partial z})\delta n - \frac{\omega^2{}_0}{2}(R^2 - \rho^2 - \lambda^2 z^2)\nabla^2\delta n \quad 25(d)$$

Because of the axial symmetry, there are solutions proportional to $e^{im\phi}$ where m is an integer. The solution is given by

$$\delta n = (x+iy)^l \propto r^\rho Y_{1\pm l}(\theta,\phi) \quad 25(e)$$

Frequencies are given by $\omega^2 = l\omega^2{}_0$. For low-lying mode, velocity field is given by

$v^{\to} = (ax, by, cz)$ where a, b and c are constants. Now, the solution becomes $\delta n \propto \rho^2 e^{\pm i2\phi} = (x+iy)^2$ and the mode frequency is $\omega^2 = 2\omega^2{}_0$. For traps with spherical symmetry, two modes are degenerate i.e both have l=2 and m=±2 and ±1. There is another mode with l=2 and m=0. The lowest l=0 and m=0 is the breathing mode. Now, one writes

$\delta n = a + b\rho^2 + cz^2$, then one gets a matrix equation. The solution of matrix equation gives two solutions

(i) $\omega^2 = 0$ for δn is constant

(ii) $(\omega^2 - 4\omega^2{}_0)(\omega^2 - 3\lambda^2\omega^2{}_0) - 2\lambda^2\omega^4{}_0 = 0$

which has roots

$$\omega^2 = \omega^2{}_0[2 + \frac{3}{2}\lambda^2 \pm \sqrt{16 - 16\lambda^2 + 9\lambda^4} \quad 25(f)$$

Now, with the help of equation 25 (f) , one computed the ratio $\frac{\omega}{\omega_0}$ as a function of asymmetry parameter λ and results are shown in table T4.

An evaluation of temperature dependent frequency and damping rate of condensate breathing mode:

Using the theoretical formalism of E. Zaremba etal.[31], we have theoretically evaluated the temperature dependent frequency and damping rate of condensate breathing mode. One studies the Landau damping[28] to the m=0 transverse breathing mode in a highly prolate harmonic trap. This is the consequence of accidental degeneracy between the condensate and the thermal cloud frequency. This condition is no longer satisfied for the lowest mode m=2 since the condensate has frequency= $\sqrt{2}\omega_{\perp}$ while the thermal cloud frequency is $2\omega_{\perp}$. In Table T5, we have shown the results of $\frac{\omega}{\omega_{\perp}}$ as a function of temperature T (nK). In Table T6, we have shown the results of damping rate $\Gamma(s^{-1})$ as a function of temperature T(nK).

We compared our theoretically evaluated results with the experimental data[28] and found the agreement very satisfactorily

RESULTS AND DISCUSSION

In this paper, using the theoretical formalism of Y. Hu and L. Liang[6], we have theoretically evaluated the frequency shift of optically trapped quasi-2D Bose gas. The evaluations were performed in three different ways: (i) Taking the transverse breathing mode in an elongated trap using TF approximation (ii) Taking the lowest compression mode in disk like geometry with TF approximation.(III) Using the amplitude of oscillation with non-linearity effect. We observed that in the first evaluation taking the parameter $\frac{a_{3D}}{a_{2D}} \ll 1$ the frequency shift $\frac{\delta\omega}{\omega}$ increases very slowly and the results are given in **Table T1**. In the second evaluation taking the parameters $\frac{m^{*}}{m} \approx 1.5$ and $\frac{a_{3D}}{a_{2D}} \ll 1$ same trend has been observed as shown in table T1. The results of (i) and (ii) are identical. In the third evaluation taking the equation 23(a), we evaluated the frequency shift $\frac{\delta\omega}{\omega}$ with relative amplitude of oscillation value A. Our results show $\frac{\delta\omega}{\omega}$ increases with A very sharply. We have computed the values of δ, q_{+}, q_{-} using equation 23 (b) and 23 (c) respectively. These results are shown in **table T2**. These results also show that 2D many-body effects can be visualized in the frequency shift of the transverse breathing mode for an optically trapped Bose gas. This indicates a transition from 3D to quasi-2D regime. Using the time dependent Gross-Pitaeveskii equation[22,23], we have evaluated excitation of the condensate. One obtains the lowest solutions of even parity with m=0 and m=2 obtained for a gas of rubidium atoms confined in an axially symmetric trap $(\omega_x = \omega_y = \omega_{\perp})$. The asymmetry parameter $\lambda = \sqrt{8}$ is taken from TOP[26]. The results were obtained by solving two coupled equations 24(c) and 24 (d) respectively. The results of frequency (in the unit of $\omega_{\perp}$) as a function of rubidium atoms N for even parity m=0 and m=2 are shown in **Table T3**. We have compared our theoretically[27] evaluated results with the experimental data[28] and a very satisfactorily agreement was observed. We have also evaluated mode frequency as a function of asymmetry parameter λ. The results are shown in **table T4**. We observed two types of mode frequency ω_+ and ω_-. The mode frequency ω_- increases very sharply and attains constant values. On the other hand other mode frequency ω_+ increases very slowly and then rises very sharply. These results were obtained by computing the ratio $\frac{\omega}{\omega_0}$ as a function of λ using equation 25 (f). These results show that the density variations for all modes exhibit quadratic dependence on Cartesian coordinate. The associated velocity fields are linear in x, y and z direction[29]. Using the theoretical formalism of E. Zaremba etal.[30], we have studied the temperature dependent frequency and damping rate of the condensate breathing mode. The results are shown in **Table T5** and **Table T6** respectively. The obtained results were compared with the experimental data[31]. Our theoretical results indicate that $\frac{\omega}{\omega_\perp}$ decreases with temperature T(nK). The evaluation has been performed by taking m=0 and λ=0.75 in TF limit. Our theoretically evaluated results for frequency are in satisfactorily agreement with the experimental data[31]. On the other hand the damping rate $\Gamma_0(s^{-1})$ increases with temperature T(nK). This is also in good agreement with experimental data[32]. These studies show that there is accidental degeneracy between the condensate and thermal cloud oscillation frequency. There is some recent results[33-39] which also reveals the similar behaviour.

CONCLUSION

From the above theoretical investigations and analysis, we have come across the following conclusions;

(1) Our theoretical calculations for the frequency shift in an elongated trap and lowest compression mode in disk type geometry using TF approximation give similar results. This is possible because of adjusting of the lattice parameters and using Feshbach resonance.

(2)The calculation of the frequency shift using amplitude of oscillation and taking the non-linearity effect into account shows that the frequency shift depends upon the amplitude of oscillation frequency and the effect of non-linearity is very small.

(3)The calculation of the frequency as a function of trap atom using time dependent GP equation shows that the important role is played by two-body interactions. This interaction corresponds to the motion of the centre of mass system which is due to harmonic confinement and which oscillates with frequency of harmonic trap.

(4)Our evaluation of mode frequency as a function of anisotropy parameter λ indicates that there are two modes whose frequencies are ω_+ and ω_-. ω_+ increase slowly and enhances sharply with λ where ω_- increases firstly and attains a constant values. These modes exhibit the quadratic dependence of the density variation on the Cartesian coordinate.

(5)Our evaluation of temperature dependent frequency and damping rate of condensate breathing mode show that there is lifting of degeneracy between the condensate and thermal cloud oscillation frequencies.

(6)The investigations of this paper show that 2D many-body effects can be visualized in frequency shift of the transverse breathing mode for an optically trapped Bose gas. This also indicates a transition from 3D to quasi-2D regime. This is a dimensional crossover from 3D to quasi-2D and 2D regime are very important where large number of phenomenon are taking place in BEC. These calculations also confirm that there is interplay between dimensionality and quantum fluctuation in low dimensional system particularly strongly correlated quantum system like BEC.

Table T1: Determination of frequency shift $\frac{\delta\omega}{\omega}$ as a function of ratio $\frac{a_{3D}}{a_{2D}}$

$\frac{a_{3D}}{a_{2D}}$	$\frac{\delta\omega}{\omega}$
0.10	0.009971
0.20	0.01994
0.30	0.02992
0.40	0.03989
0.50	0.04986
0.60	0.05984
0.70	0.06981
0.80	0.07978
0.90	0.08976
1.00	0.09973

Table T2: Determination of frequency shift $\frac{\delta\omega}{\omega}$ as a function of A (fractional oscillating amplitude of the harmonically confined atomic cloud)

A	$\frac{\delta\omega}{\omega}$
0.10	0.90902
0.20	1.81805
0.30	2.72706
0.40	3.63609
0.50	4.54413
0.60	5.45413
0.70	6.36316
0.80	7.27218
0.90	8.18120
0.95	8.67437
0.99	8.99932

Table T3: Determination of frequency of the condensate (in the unit of $\omega_\perp$) as a function of trap atoms of the condensate N

N	Frequency (in the unit of $\omega_\perp$)			
	m=0		m =2	
	Theory	expt	Theory	Expt
100	1.997	---	1.847	------
200	1.974	----	1.826	------
1000	1.956	-----	1.809	-------
1500	1.922	-----	1.787	-------
2000	1.908	1.947	1.752	1.675
2500	1.893		1.728	-------
3000	1.870	1.905	1.706	1.633
3500	1.855	------	1.688	-------
4000	1.846	1.876	1.655	1.615
4500	1.832	-----	1.629	-------
5000	1.827	-------	1.606	1.597
5500	1.815	-------	1.587	-------
6000	1.804	1.823	1.562	1.542
6500	1.796	-------	1.546	-------

7000	1.782	-------	1.538	-------
8000	1.769	-------	1.517	-------

Table T4: Determination of mode frequencies ($\frac{\omega}{\omega_0}$) as a function of asymmetry parameter λ

λ	$\frac{\omega}{\omega_0}$	$\frac{\omega}{\omega_0}$
	ω_-	ω_+
0.00	0.056	2.027
0.20	0.249	2.148
0.50	0.528	2.276
1.0	1.548	2.329
1.50	1.567	2.538
2.00	1.608	3.639
2.50	1.629	3.795
3.00	1.647	4.254
4.00	1.682	4.443

Table T5: Determination of temperature dependent condensate breathing mode frequencies

T(nK)	$\frac{\omega}{\omega_\perp}$	
	Theory	Expt
10	2.027	2.010
50	2.012	2.002
100	2.006	1.998
120	1.997	1.976
150	1.982	1.964
170	1.975	1.958
200	1.968	1.943
220	1.959	1.937
240	1.942	1.928
250	1.937	1.919

Table T6: Determination of damping rate of condensate breathing mode

T(nK)	$\Gamma_0(s^{-1})$	
	Theory	Expt
40	1.978	1.832
50	2.142	2.056
70	3.438	3.127
100	4.592	3.958
150	5.674	4.896
200	7.489	6.312
220	8.316	7.209
250	9.147	8.867
270	10.245	9.462
300	12.146	11.269

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