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A THEORETICAL STUDY OF FERMIONIC CONDENSATION IN ULTRA COLD ATOMS AND NUCLEAR MATTER.

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Abstract: - Using the theoretical formalism of **Armen Sedrakian and John W. Clark [Phys. Rev C73, 035803 (2006)],** we have studied pair condensation and bound states in fermionic systems. We evaluated the gap function and chemical potential as a function of temperature and observed that in the density-temperature domain there is a crossover from BCS condensate of Cooper pairs to BEC condensate of tightly bound deuterons. In an attractive fermion system, one studies the two-body (dimmer) and three-body (trimmers) bound states in free space. We observed that at high temperature/low density the system is populated by trimmers whereas low temperature/high density system supports the condensate of neutron-proton Cooper pairs. This study is quite helpful in order to understand the interaction characteristics of nuclear system. Using the theoretical formalism of **Luca Salasnich [Journal of Physics: Conference series 497, 012026 (2014)],** we have studied the fermionic condensation in nuclear matter and neutron star. Our theoretical evaluated result of condensate fraction (n_0/n) of neutron pairs in neutron matter as a function of neutron- number density (n/n_S) , n_S is nuclear saturation density indicates that condensate fraction increase and attains maximum and then decreases. Our evaluated results of condensate fraction (n_0/n) as a function of scaled distance (r/R) for 1.4 solar mass neutron star show that the maximum condensate fraction exists in the crust of neutron star. Our evaluated theoretical results are in good agreement with the other theoretical workers.

Keywords: Pair condensation, two-body bound state (dimmer), three-body bound state (trimmers), neutron-proton Cooper pairs, BEC condensate of deuterons, neutron matter, neutron star.

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INTRODUCTION

In this paper using the theoretical formalism of A. Sedrakian and John W. Clark[1], we have theoretically studied the pair condensation and bound states in fermionic systems. We have studied the finite temperature density phase diagram of an attractive fermionic system that support two-body (dimer) and three-body (trimer) bound state in free space. Using the interaction characteristic for nuclear matter, we have evaluated the pairing gap as a function of temperature for the fixed values of the ratio $f=n_0/n$, where ni the baryon density and no is the satuaration density of symmetrical nuclear matter. The chemical potential is also evaluated as a function of temperature with fixed values of diluteness parameter na^3 where a is scattering length. We have also evaluated the dependence of two-body and three-body binding energies E_d and E_f respectively as a function of inverse temperature for the fixed values of the ratio $f=n_0/n$ for dilute nuclear matter. Our theoretical results indicate that the binding energies of two and three body bound states depend upon the choice of interaction. We have also evaluated the critical temperatures for the super fluid phase transition and for extinction of three-body bound states as a function of density. Our theoretical obtained results show that low temperature and low-density domain [na^3 <<1, f<40) contains a Bose condensate of a tightly bound deuterons. The low-temperature, high density domain features a BCS condensate of weakly bound Cooper pairs (na^3>>`1). The domain between these two characteristics contains nucleonic liquid. Our theoretically obtained results are in good agreement with the other theoretical workers[2-4].

In another work, using the theoretical formalism of Luca Salasnich[5], we have investigated the Bose-Einstein condensation of fermionic pairs in three different super fluid systems. The system includes ultra cold and dilute atomic gases, bulk neutron matter and neutron stars. Here, we have determined the condensate fraction of fermionic atoms as a function of the inverse interaction strength ($1/k_Fa$), k_F is Fermi wave number. Our theoretically evaluated results are in good agreement with other theoretical workers[6,7].

MATERIALS AND METHODS

Evaluation of Pairing gap and chemical potential as a function of temperature

In order to evaluate pairing gap and chemical potential as a function of temperature, one has to solve the gap equation. One starts with real time Green's function formalism in which the propagator are assumed to be ordered on the Schwinger-Keldysh real time contour[8]. The correlation function and self-energies are in the form of 2x2 matrices. The one-body Green's function is defined in terms of the fermionic fields $\psi(x)$

$$G_{\alpha\beta}(x,x') = \begin{pmatrix} G^{c}{}_{\alpha\beta}(x,x'), & G^{<}{}_{\alpha\beta}(x,x') \\ G^{>}{}_{\alpha\beta}(x,x'), & G^{a}{}_{\alpha\beta}(x,x') \end{pmatrix}$$

$$= \begin{pmatrix} \langle T^{c}\psi^{\dagger}{}_{\alpha}(x)\psi_{\beta}(x')\rangle & \langle \psi^{\dagger}{}_{\alpha}(x)\psi_{\beta}(x')\rangle \\ \langle \psi^{\dagger}{}_{\alpha}(x)\psi_{\beta}(x')\rangle & \langle T^{a}\psi^{\dagger}{}_{\alpha}(x)\psi_{\beta}(x')\rangle \end{pmatrix} \quad (1)$$

Here, $T^{c/a}$ are the time-ordering and anti-ordering operators. $\langle\ \rangle$ stands for statistical averaging over the equilibrium grand-canonical ensemble and x is space-time four vectors. The Greek indices stand for discrete quantum numbers (spin ,isospin). In, equilibrium, the physical properties of the system is described by the retarded propagator

$$G^{R}{}_{\alpha\beta}(x,x') = \theta(t-t')[G^{>}{}_{\alpha\beta}(x,x') - G^{<}{}_{\alpha\beta}(x,x')] \quad (2)$$

where $\theta(t)$ is the step function. The retarded and advanced propagators are also related to the elements of the Schwinger-Keldysh matrix (1) through a rotation in the matrix space by the unitary operator $U = (1+i\sigma_y)/\sqrt{2}$, where σ_y is the y-component of the vector of Pauli matrices. The 4x4 matrix Green's function satisfies the familiar Dyson equation

$$G_{\alpha\beta}(x,x') = G_{\alpha\beta}{}^{0}(x,x') + \Sigma_{\gamma,\delta}\int d^4x''d^4x'''$$
$$xG_{\alpha\beta}{}^{0}(x,x''')\Sigma_{\gamma,\delta}(x''',x'')G_{\alpha\beta}(x'',x') \quad (3)$$

where free propagators $G_{\alpha\beta}{}^{0}(x,x')$ are diagonal in the Gor'kov space. Taking the Fourier transformation of equation (3) , one gets

$$G_{\alpha\beta}(p) = G_{0\alpha\beta}(p) + G_{0\alpha\gamma}(p)x[\Sigma_{\gamma\delta}(p)\, G_{\delta\beta}(p) + \Delta_{\gamma\delta}(p)Fd_{\delta\beta}(p)] \quad (4)$$

$$F^{\dagger}{}_{\alpha\beta}(p) = G_{0\alpha\gamma}(-p)x[\Delta^{\dagger}{}_{\gamma\delta}(p)G_{\delta\beta}(p) + \Sigma_{\gamma\delta}(-p)\,F\Delta_{\delta\beta}(p)] \quad (5)$$

Here, p is the four momentum, $G_{0\alpha\beta}(p)$ is the free normal propagator and $\Sigma_{\alpha\beta}(p)$ and $\Delta_{\alpha\beta}(p)$ are normal and anomalous self-energies. $G_{\alpha\beta}(p)$ and $F_{\alpha\beta}(p)$ are two-point correlation functions. The solutions of equation (4) and (5) are obtained under quasi-particle approximation which keeps only the pole part of the propagators are

$$G^{R}{}_{\pm} = u^{2}{}_{p}(\omega - \omega_{\pm} + i\eta)^{-1} + v^{2}{}_{p}(\omega - \omega_{\pm} + i\eta)^{-1} \quad (6)$$

$$F^{R} = F^{\dagger} = u_{p}v_{p}[(\omega - \omega_{\pm} + i\eta)^{-1} - (\omega - \omega_{\pm} + i\eta)^{-1} \quad (7)$$

$$G^{R}{}_{\pm} = v^{2}{}_{p}(\omega - \omega_{\pm} + i\eta)^{-1} + u^{2}{}_{p}(\omega - \omega_{\mp} + i\eta)^{-1} \quad (8)$$

Here, the quasi-particle spectrum $\omega_{\pm} = \pm\sqrt{E(p)^{2} + \Delta^{2}(p)}$ and the Bogolyubov amplitudes u_p and v_p are normalised. $u^{2}{}_{p} = 1/2 + E(p)/2\omega_{\pm}$ and $u^{2}{}_{p} + v^{2}{}_{p} = 1$. In equilibrium the elements of the 2x2 matrix appearing in equation (1) are determined from the retarded and advanced propagators as

$$G^{<}(p) = [G^{A}(p) - G^{R}(p)]f(\omega)$$

$$G^{>}(p) = G^{R}(p) + G^{<}(p) \quad 9(a)$$

$$G^{<}(p) = G^{<}(p) + G^{R}(p)$$

$$G^{a}(p) = G^{<}(p) - G^{A}(p) \quad 9(b)$$

where $f(\omega)$ is the Fermi distribution function. For time-local approximations, both the pairing interaction and the pairing gap are energy independent. The mean-field approximation to the anomalous self-energy (the gap-function) is then

$$\Delta^R(p) = 2\int \frac{d\omega dp'}{(2\pi)^4} V(p^{\rightarrow}, p^{\rightarrow}{}') \operatorname{Im} F^R(\omega, p^{\rightarrow}{}') \, f(\omega) \quad (10)$$

Substituting equation (7), one can obtain two-coupled integral equations for the gap (l=0, 2)

$$\Delta_l(p) = -\int \frac{dp' p'^2}{(2\pi)^2} \Sigma_l V^{3SD1}{}_{ll'}(p, p') \frac{\Delta_{l'}(p')}{\sqrt{E(p)^2 + D(p')^2}}$$

$$x[f(\omega_+) - f(\omega_-), (l, l' = 0, 2) \quad (11)$$

where $D^2(k) = (\frac{3}{8\pi})[\Delta^2{}_0(k) + \Delta^2{}_2(k)]$ is the angle-averaged neutron-proton gap function and $V^{3SD1}(p, p')$ is the interaction in the $3S1 - 3D1$ channel (the dominant attractive channel in dilute and isospin-symmetric nuclear matter). The chemical potential is then determined self-consistently from the gap equation (11) and the expression for the density is given by

$$n = -8\int \frac{dp^{\rightarrow} d\omega}{(2\pi)^4} \operatorname{Im} G^R{}_+(\omega, p^{\rightarrow}) f(\omega)$$

$$= 4\int \frac{d^3p}{(2\pi)^3} [u^2{}_p f(\omega_+) + v^2{}_p f(\omega_-)] \quad (12)$$

The factor 4 comes from the sum over the two projections of spin and isospin.

<u>Two-body bound states:</u>

Now, one considers temperature above the critical temperature of pair production. The two-body T matrix that sums up the particle –particle ladders for a system interacting with the potential V obeys the operator equation

$T=V+VG_0T$ $=V+TG_0V$ (13)

Since the potential is time local, the T-matrix depends on two-time arguments and its transformation properties are identical to those of the two-point correlation function. The four momenta p=(E, **P)** and $p=(\varepsilon,\vec{p})$ in the center –of-mass system are related to their counterparts $k_{1,2}=(\omega_{1,2},\vec{k}_{1,2})$ in the laboratory system through $P=k_1+k_2$ and $p=(k_1-k_2)/2$.

In the momentum representation, equation (13) takes the form

$$T^R(\vec{p},\vec{p}\,',\vec{P},E)=V(\vec{p},\vec{p}\,')+\int\frac{d\vec{p}\,''}{(2\pi)^3}\,V(\vec{p},\vec{p}\,'')\,x\,G^R{}_0(\vec{p}\,'',\vec{P},E)\,T^R(\vec{p}\,'',\vec{p};\vec{P},E) \qquad (14)$$

for the retarded component of the T-matrix. The relevant two-body Green's function is given by

$$G^R{}_0(\vec{k}_1,\vec{k}_2,E)=\int_{\omega_1\omega_2}\frac{G^>(k_1)G^>(k_2)-G^<(k_1)G^<(k_2)}{E-\omega_1-\omega_2-i\eta}$$

$$=\frac{Q_2(\vec{k}_1,\vec{k}_2)}{E-\varepsilon(\vec{k}_1)-\varepsilon(\vec{k}_2)+i\eta} \qquad (15)$$

In the second relation, one has used the quasi-particle approximation. One has $f_\omega=\int\frac{d\omega}{2\pi}$,

The two-body phase space occupation factor $Q_2(\vec{k}_1,\vec{k}_2)=1-f(\vec{k}_1)-f(\vec{k}_2)$ is operating in intermediate states and allowing the propagation of particle and holes, thereby incorporating time-reversal invariance. The two-body T^R-matrix has a pole at the energy corresponding to the two-body bound state. If Q_2 =1, the pole is exactly at the binding energy of the deuteron; otherwise the pole on the real energy axis determines the binding energy of a dimmer in the background medium of finite density and temperature.

Three-body bound states

In this case the three-body equation is given by

$$T=V+VGV=V+G_0T \qquad (16)$$

Here T is three –body matrix and V is three-body interaction and G and G_0 are full and free three-body Green's function. The three-body propagator G_0 is written in the momentum representation

$$G_0(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}, \Omega) = \int_{\omega_1,\omega_2,\omega_3} [G^>(k_1) G^>(k_2)G^>(k_3) - G^<(k_1)G^<(k_2)G^<(k_3)]$$

$$= \frac{Q_3(k_1,k_2,k_3)}{\Omega - \varepsilon(k_1) - \varepsilon(k_2) - \varepsilon(k_3) + i\eta} \quad (17)$$

Here, $k_i = (\omega_i, \vec{k}_i)$ is the particle four-momenta and $Q_3(k_1,k_2,k_3)$ is the intermediate –state phase-space occupation factor for three particle propagation and is given by

$$Q_3(k_1,k_2,k_3) = [1 - f(k_1)][1 - f(k_2)[1 - f(k_3) - f(k_1)f(k_2)f(k_3)$$

(18)

These equations are solved in the form of integral equations in the states diagonal in the angular momentum basis

$$|pq\alpha\rangle_i = |pq(l\lambda)LM(s1/2)SM_s\rangle_i \quad (19)$$

Where p and q are the magnitudes of the relative momenta of the pair (kj), l and λ are their associated relative angular –momentum quantum numbers, s is total spin and $LMSM_S$ are the orbital and spin quantum numbers of the three-body system. The two-body T-matrix is given by

$$\langle p|T(\omega)|p'\rangle = \langle p|V|p'\rangle + \int \frac{dp''p'^2}{4\pi^2} \langle p'|V|p''\rangle$$

$$X \frac{Q_2(p,q)}{\omega - \varepsilon_+(p,q) - \varepsilon_-(p,q) + i\eta} \langle p''|T(\omega)|p'\rangle \quad (20)$$

Where $Q_2(q,p) = \langle 1 - f(q/2+p) - f(q/2+p)\rangle$ and $\varepsilon_\pm(q,p) = \langle \varepsilon(q/2 \pm p)\rangle$ are averaged over the angle between the vectors **q** and **p.** The three-body propagator in the $|pq\alpha\rangle_i$ has the form

$$\langle pq\alpha|G_0\Omega|p'q'\alpha'\rangle = \delta_{\alpha\alpha'}\frac{\delta(p-p')}{p^2}\frac{\delta(q-q')}{q^2}$$

$$x\frac{Q_3(q,p)}{\Omega-\varepsilon_+(q,p)-\varepsilon_-(q,p)-\varepsilon(-q)} \quad (21)$$

Where Q_3 (q,p) is given by equation (18). Here, the propagator is assumed to be independent of the momentum of the three-body system with respect to the background (**K =0)**. Finally , the required expression for the permutation operator P in the chosen basis

$$\langle pq\alpha|P|p'q'\alpha'\rangle = \int_{-1}^{+1} dx \frac{\delta(\pi_1-p)}{p^{l+2}}\frac{\delta(\pi_2-p')}{(p')^{l'+2}}\, xH_{\alpha\alpha'}(q,q',x) \quad (22)$$

Here, $\pi^2{}_1 = q'^2 + q^2/4 + qq'x$, $\pi^2{}_2 = q^2 + q'^2/4 + qq'x$ and x is the angle formed by **q** and **q'** $H_{\alpha\alpha'}(q,q'x)$ is given by

$$H_{\alpha\alpha'}(q,q'x) = \sum_{n=0}^{\infty} P_n(x)\Sigma_{l_1+l_2=l}\Sigma_{l'_1+l'_2=l'}\, q^{l_2+l'_2}(q')^{l_2+l'_2}\, h^{nl_1l'_1l_2l'_2}{}_{\alpha\alpha'} \quad (23)$$

Here, P_n (x) is Legendre polynomial and $h^{nl_1l'_1l_2l'_2}{}_{\alpha\alpha'}$ is coefficients of combination of 3j and 6j symbols[9]. The resulting equation is solved with the method of iteration[10].

Evaluation of condensate fraction n_0/n of neutron pairs in neuclear matter and neutron star

In order to evaluate condensate fraction n_0/n of neutron matter and neutron star, one takes help of the formalism of Luca Salasnich[5], which studies the fermionic condensation in ultra cold atoms, nuclear matter and neutron stars.

A quantum system of interacting identical bosons can be described by the bosonic field operator which satisfies the familiar commutation relation[11]

$$[\Phi(r^{\rightarrow}),\Phi^{\dagger}(r'^{\rightarrow})] = \delta(r^{\rightarrow}-r^{\rightarrow}{}') \quad 24(a)$$

$$[\Phi^{\dagger}(r^{\rightarrow}),\Phi^{\dagger}(r^{\rightarrow}{}')] = [\Phi(r^{\rightarrow}),\Phi(r^{\rightarrow}{}')] = 0 \quad 24(b)$$

The bosonic one-body density matrix is given by

$$n(\vec{r},\vec{r}\,') = \left\langle \Phi^{\dagger}(\vec{r})\Phi(\vec{r}\,') \right\rangle \quad 24(c)$$

Here $\delta(\vec{r})$ is the Dirac-delta function. $\langle\ \rangle$ is a thermal average. The total number of bosons is given by

$$N = \int \left\langle \Phi^{\dagger}(\vec{r})\Phi(\vec{r}) \right\rangle d^3\vec{r} \quad 24(e)$$

The condensate number N_0 of bosons are given by

$$N_0 = \int \left| \left\langle \Phi(\vec{r}) \right\rangle \right|^2 d^3\vec{r} \quad 24(f)$$

Similarly for quantum system of interacting identical fermions with two spin components $\sigma = (\uparrow,\downarrow)$ can be described by the fermionic field operator $\psi_\sigma(\vec{r})$ which satisfies the familiar anti-commutation rules

$$\{\psi_\sigma(\vec{r}), \psi^{\dagger}_{\sigma'}(\vec{r}\,')\} = \delta(\vec{r} - \vec{r}\,')\delta_{\sigma\sigma'} \quad 25(a)$$

$$\{\psi^{\dagger}_\sigma(\vec{r}), \psi^{\dagger}_{\sigma'}(\vec{r}\,')\} = \{\psi_\sigma(\vec{r}), \psi_{\sigma'}(\vec{r}\,')\} = 0 \quad 25(b)$$

Here, $\delta_{\sigma\sigma'}$ is Kronecher delta. The fermionic one-body density matrix is given by

$$n_{\sigma,\sigma'}(\vec{r},\vec{r}\,') = \left\langle \psi^{\dagger}_\sigma(\vec{r})\psi_{\sigma'}(\vec{r}) \right\rangle \quad 25(c)$$

The total number of fermions read as

$$N = \Sigma_{\sigma=\uparrow,\downarrow} \int \left\langle \psi^{\dagger}_\sigma(\vec{r})\psi_\sigma(\vec{r}) \right\rangle d^3\vec{r} \quad 25(d)$$

Number of condensed fermions N_0 is given by

$$N_0 = 2\Sigma_{\sigma,\sigma'=\uparrow,\downarrow}\int\left|\left\langle \psi_\sigma(\vec{r})\psi_{\sigma'}(\vec{r}')\right\rangle\right|^2 d^3\vec{r}\, d^3\vec{r}' \qquad 25(e)$$

The shifted Hamiltonian of the uniform two-spin component Fermi super fluid made of ultra cold atoms is given by

$$H = \int d^3\vec{r}\,\Sigma_{\sigma=\uparrow\downarrow}\psi^\dagger{}_\sigma(\vec{r})[-\frac{\hbar^2}{2m}\nabla^2 - \mu]\,\psi_\sigma(\vec{r})$$
$$+g\psi^\dagger{}_\uparrow(\vec{r})\psi^\dagger{}_\downarrow(\vec{r})\psi_\downarrow(\vec{r})\psi_\uparrow(\vec{r}) \qquad (26)$$

Where $\psi_\sigma(\vec{r})$ is the field operator that annihilates a fermion of spin σ in the position **r**, while $\psi^\dagger{}_\sigma(\vec{r})$ creates a fermion of spin σ in **r.** Here g<0 is the strength of the attractive fermion-fermion interaction which is approximated by a contact Fermi pseudo-potental[12]. Because of ultra cold and dilute gases the average distance between atoms is much larger than the effective radius of the inter-atomic potential. The total number N is fixed by the chemical potential μ which appears in equation (26)

Within the Bogoliubov approach the mean field Hamiltonian shown in equation (26) can be diagonalized by using the Bogoliubov-Valatin representation[13] to the field operator $\psi_\sigma(\vec{r})$ in terms of the anti-commuting quasi-particle Bogoliubov operator $b_{\vec{k}\sigma}$ with amplitudes $u_{\vec{k}}$, $v_{\vec{k}}$ and the quasi-particle energy $E_{\vec{k}}$. The expressions of these parameters are given by

$$E_{\vec{k}} = [(\varepsilon_{\vec{k}} - \mu)^2 + \Delta^2]^{\frac{1}{2}} \qquad 27(a)$$

$$u^2{}_{\vec{k}} = (1 + (\varepsilon_{\vec{k}} - \mu)/E_{\vec{k}})/2 \qquad 27(b)$$

$$v^2{}_{\vec{k}} = (1 - (\varepsilon_{\vec{k}} - \mu)/E_{\vec{k}})/2 \qquad 27(c)$$

$$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m} \qquad 27(d)$$

$$\frac{-1}{g}=\frac{1}{\Omega}\Sigma_{k\rightarrow}\frac{1}{2E_{k\rightarrow}} \quad 27(e)$$

$$n=\frac{2}{\Omega}\Sigma_{k\rightarrow}v^2{}_{k\rightarrow} \quad 27(f)$$

Here, $\varepsilon_{k\rightarrow}$ is the single-particle energy, Δ is the pairing gap and Ω is the volume of the uniform system. N is the total density of the fermions. The condensate density of paired fermions is given by

$$n_0=\frac{2}{\Omega}\Sigma_{k\rightarrow}u^2{}_{k\rightarrow}v^2{}_{k\rightarrow} \quad 28(a)$$

With the help of parameters u_k, v_k, μ and Δ the above equation assumes the form

$$n_0=\frac{m^{\frac{3}{2}}}{8\pi\hbar^3}\Delta^{\frac{3}{2}}\sqrt{\frac{\mu}{\Delta}+\sqrt{1+\frac{\mu^2}{\Delta^2}}} \quad 28(b)$$

Nuclear Matter

For the study of nuclear matter, one takes help of the following Hamiltonian[14]

$$H=\int d^3r^{\rightarrow}\Sigma_{\sigma=\uparrow,\downarrow}\psi^{\dagger}{}_{\sigma}(r^{\rightarrow})[-\frac{\hbar^2\nabla^2}{2m}-\mu]\,\psi_{\sigma}(r^{\rightarrow})$$
$$+\int d^3r^{\rightarrow}d^3r^{\rightarrow'}\psi^{\dagger}{}_{\uparrow}(r^{\rightarrow})\psi^{\dagger}{}_{\downarrow}(r^{\rightarrow'})V(r^{\rightarrow}-r^{\rightarrow'})\,\psi_{\downarrow}(r^{\rightarrow'})\psi_{\uparrow}(r^{\rightarrow}) \quad 28(b)$$

Where $\psi_{\sigma}(r^{\rightarrow})$ is the field operator that annihilates a neutron of spin σ in the position **r**, while $\psi^{\dagger}{}_{\sigma}(r^{\rightarrow})$ creates a neutron of spin σ in **r.** Here. $V(r^{\rightarrow}-r^{\rightarrow'})$ is the neutron-neutron potential characterized by s-wave scattering length a=-18.5 fm and effective range[15]=2.7fm. Now, applying the familiar Bogoliubov approach, equation 28(b) can be written in the integral form

$$\Delta_{\vec{q}} = \Sigma_{\vec{k}} V_{\vec{q}\vec{k}} \frac{\Delta_{\vec{k}}}{2E_{\vec{k}}} \quad 28(c)$$

Where $$V_{\vec{q}\vec{k}} = \langle \vec{q}, -\vec{k} | V | \vec{k}, -\vec{k} \rangle \quad 28(d)$$

This is the wave-number representation of the neutron-neutron potential. $E_{\vec{k}}$ is given by

$$E_{\vec{k}} = \sqrt{(\frac{\hbar^2 k^2}{2m} - \mu)^2 + |\Delta_{\vec{k}}|^2} \quad 28(e)$$

Under the simplifying assumption, $\mu \approx \varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}}$ and $\Delta_{\vec{k}} \approx \Delta$, one determines the condensate fraction[16]

$$\frac{n_0}{n} = \frac{\pi}{2^{\frac{5}{6}}} \frac{\sqrt{\frac{\varepsilon_F}{\Delta} + \sqrt{1 + \frac{\varepsilon_F^2}{\Delta^2}}}}{I_2(\frac{\varepsilon_F}{\Delta})} \quad 28(f)$$

Where $$I_2(\frac{\varepsilon_F}{\Delta}) = I_2(x) = \int_0^\infty y^2 (1 - \frac{(y^2 - x)}{\sqrt{(y^2 - x)^2 + 1}}) dy \quad 28(g)$$

In the deep BCS regime, where $\frac{\Delta}{\varepsilon_F} \ll 1$, one finds

$$\frac{n_0}{n} = \frac{3\pi}{8} \frac{\Delta}{\varepsilon_F} \quad 28(h)$$

Now, using the numerical data of $\frac{\Delta}{\varepsilon_F}$ as a function of ε_F obtained [17] from realistic neutron-neutron potentials, one gets

$$\frac{\Delta}{\varepsilon_F} = \frac{\beta_0 k^{\beta_1}{}_F}{\exp(k^{\beta_2}{}_F / \beta_3) - \beta_3} \quad 28(I)$$

Now, using the parameters $\beta_0 = 2.851, \beta_1 = 1.942, \beta_2 = 1.657$ and $\beta_3 = 0.274$ with the help of equation 28(f), one can get the condensate fraction of neutron matter as a function of neutron density.

Neutron stars

Neutron stars are astronomical compact objects which can result from the gravitational collapse of massive star during supernova event. Such stars are mainly composed of neutrons. Neutron stars are very hot and are supported against further collapse by Fermi pressure. A typical neutron star has a mass M between 1.35 and 2.0 solar masses with a corresponding radius R of about 12 km. The crust of neutron star s is super fluid with temperature T=10^8K. In this paper, we have studied the condensate fraction n_0/n of the neutron star as a function of distance r. The results are shown in **table T7 and T8** respectively

RESULTS AND DISCUSSION

Using the theoretical formalism of A Sedrakian and John W Clark[1], we have studied pair condensation and bound states in fermionic systems. This formalism studies the finite temperature density phase diagram of an attractive fermionic system that supports two-body (dimmer) and three-body (trimmers) bound states in free space. Pairing correlation and three-body bound states are universal properties of attractive fermions that are considerable interest in number of fields. Trapping and manipulating of cold fermionic atoms has opened a new window on the many-body properties of dilute Fermi system[18]. The possibility of manipulating the strength of the interactions in these systems by tuning a Feshbach resonances allow one to explore the phase diagram in different regions particularly crossover in a controlled manner. **In table T1,** we have shown the evaluated result of pairing gap Δ[MeV] as a function of temperature T[MeV] for fixed values of ratio f=(n_0/n). Here n is the baryon density and n_0 =0.16fm^{-3} the saturation density of symmetrical nuclear matter. We have evaluated the result for f=20, 40, 60, 80 and 150. Our obtained results indicate that Δ decrease with T for all the values of f. The result is maximum for f=20 and minimum for f=150. In **table T2,** we have presented an evaluated result of chemical potential μ [MeV] as a function of T[MeV] for different values of na^3. na^3 is diluteness parameter with scattering length a =5.4 fm. Our obtained results show that μ increase very slowly as a function of T for each value of na^3. The value is large for na^3 =1.25 and small for na^3 =0.15. **In table T3**, we have shown the evaluated results of two-body binding energy (E_d) and three-body binding energy (E_t) as a function of inverse temperature $\beta[\mathrm{MeV}]^{-1}$ for fixed value of the ratio f=(n_0/n). This results show the temperature dependence of the two-body and three-body bound states energies in dilute nuclear matter for several values of density of environment. Our theoretically obtained results also show that the ratio $E_t(\beta)/E_d(\beta)$ is a universal constant independent of temperature. **In table T4,** we have presented the evaluated results of critical temperature T [MeV] of three-body bound states as a function of density na^3. Our theoretically obtained results show that at low density, high-temperature domain is populated by trimmers. The low temperature and low density domain (na^3<<1, f<40) contain BEC of tightly bound deuterons. The low-temperature high density domain features a BCS condensate of weakly-bound Cooper pairs (na^3>>1). The low density and low-temperature domain is populated by dimmers. The total results illustrate the scenario that the phase diagram of low-density finite temperature nuclear matter is two-fold. The system supports liquid-gas and super fluid phase transition. **In table T5,** we have shown the evaluated results of condensate fraction (n_0/n) of fermionic atoms as a function of inverse interaction strength $\frac{1}{k_F a}$. The results are performed using theoretical formalism of Luca Salasnich[5]. Mean-field results were compared with fixed mode diffusion Monte Carlo simulation results[19,20]. Our results indicate that the two theoretical approaches in the BEC side of the crossover**. In table T6,** we have presented an evaluated results of condensate fraction (n_0/n) of neutron pairs in neutron matter as a function of scaled neutron number density (n/n_S) where n_S is the nuclear saturation density. Two results were obtained by solving equation 28(f) 28(I)(Result I) and solving equation 28(h) and 28 (I) (Result II) respectively. Our theoretically obtained results indicate that at very low neutron density n, the neutron matter behaves like quasi-ideal Fermi gas with weakly correlated Cooper pairs and the condensate fraction (n_0/n) is very small. By increasing the neutron density, the attractive tail of neutron-neutron potential becomes relevant and the condensate fraction (n_0/n) grows significantly. (n_0/n) becomes maximum at (n/n_S)=10. On further increasing the density n, the repulsive core of the neutron-

neutron potential plays an important role and it destroys the correlation of Cooper pairs and condensate fraction (n_0/n) starts decreasing. Results I and II are very much identical. **In table T7,** we have shown the evaluated results of scaled density profile (n/n_S) as a function of (r/R) for 1.4 solar mass neutron star. Here n_S is nuclear saturation density and =0.16fm^{-3}. R is the radius of the star. Results were compared with PAL model[21] and Walacka model[22] of bulk neutron matter. Both results are very much identical. **In table T8,** we have presented the evaluated results of condensate fraction (n_0/n) of neutron pairs as a function of scaled distance (r/R). Obtained results were compared with two formalism results Result I and Result II. These results were obtained by solving equation 28 (f) and 28 (I) and equation 28(h) and 28 (I) respectively. These results indicate that how condensate fraction appears in the crust of the neutron star[23]. There is some recent calculations[24-28] which also reveals the similar behaviour.

CONCLUSION

From the above theoretical investigations and analysis, we have come across the following conclusions:

(1)We have studied pair condensation and bound states in fermionic systems using temperature-density phase diagram of dilute isospin –symmetric nuclear matter which features an isospin singlet and spin triplet pair condensate at low temperature and a gas of trimmers (three-body bound systems) at high temperatures.

(2)We have evaluated gap function and chemical potential by solving the gap equation. We observed that the behaviour of the system can be quantified in the density-temperature domain where the Cooper pair condensate crosses over to BEC condensate of tightly bound deuterons.

(3)We also observed that the ratio of the temperature dependent binding energies of dimmers and trimmers are independent of the temperature and density and can be determined from its value in free space. At high temperature/low density, the system is populated by trimmers whereas in low temperature/high density, the system supports a condensate of neutron-proton Cooper pairs. This then crossovers to BEC condensate of deuterons as the density decreases..

(4)We have also studied fermionic condensate in nuclear matter and neutron star. We observed that the condensate fraction in ultra cold gases of fermionic atoms can be measured with the help of momentum distribution of pairs. The condensation of fermionic pairs are analysed as a function of s-wave scattering length using the technique of Feshbach resonances. One observes small and negative value for BCS regime of Cooper Fermi pairs to small and positive value for BEC of molecular dimmers. It crosses the unitarity limit where the scattering length diverges.

(5)Our theoretically obtained results show that maximum condensate fraction exists in the crust of neutron star.

Table T1: An evaluated result of pairing gap Δ (MeV) as a function of temperature T(MeV) for fixed value of ratio $f=n_0/n$, where n is the baryon density and $n_0 = 0.16 fm^{-1}$ is the saturation density of symmetrical nuclear matter.

T [MeV]	<-------- Δ [MeV]---------------------→				
	f=20	f=40	f = 60	f =80	f= 150
0.00	5.625	4.757	3.886	3.247	2.876
0.25	5.607	4.586	3.255	3.025	2.607
0.50	4.528	3.978	3.059	2.976	2.515
0.75	4.482	3.295	2.927	2.734	2.302
1.00	4.326	3.106	2.656	2.575	1.889
1.25	3.887	2.889	2.475	2.367	1.753
1.50	3.248	2.627	2.307	2.185	1.619
1.75	2.976	2.435	2.795	2.057	1.505
2.00	2.808	2.056	2.626	1.826	1.324
2.25	2.507	1.793	2.405	1.678	1.108
2.50	2.319	1.586	2.157	1.455	1.059
3.00	1.756	1.429	1.842	1.276	0.958
3.50	1.358	1.305	1.610	1.108	0.732
4.00	1.275	1.216	1.326	1.056	0.638

Table T2: An evaluated result of chemical potential μ [MeV] as a function of temperature T [MeV] for fixed value of diluteness parameter na^3, taking scattering length a=5.4fm.

T[MeV]	<--------- μ [MeV]---------------->				
	na^3=1.25	na^3=0.65	na^3=0.30	na^3=0.20	na^3=0.15
0.00	2.953	1.542	0.325	-0.246	-0.486
0.25	2.976	1.567	0.346	-0.225	-0.472
0.50	2.998	1.594	0.355	-0.214	-0.465
0.75	3.102	1.622	0.367	-0.208	-0.457
1.00	3.147	1.645	0.376	-0.196	-0.448
1.25	3.198	1.657	0.382	-0.185	-0.432
1.50	3.226	1.678	0.385	-0.176	-0.425
1.75	3.245	1.694	0.386	-0.168	-0.416
2.00	3.276	1.712	0.390	-0.156	0.402
2.25	3.324	1.735	0.391	-0.147	-0.398
2.50	3.345	1.755	0.392	-0.132	-0.380
3.00	3.357	1.767	0.394	-0.125	-0.377
3.50	3.368	1.784	0.395	-0.116	-0.368
4.00	3.392	1.796	0.397	-0.108	-0.339

Table T3: An evaluate result of two-body binding energy (E_d) and three-body binding energy (E_t) as a function of inverse temperature $\beta[\mathrm{MeV}]^{-1}$ for the fixed value of ratio $f=n_0/n$, n is the baryon density and n_0 =0.16 fm^{-3}, n_0 is the saturation density.

$\beta[\mathrm{MeV}]^{-1}$	E_d [MeV]		E_t [MeV]	
	f =40	f =80	f =40	f =80
0.00	-7.468	-7.296	-1.867	-2.768
0.05	-6.276	-7.105	-1.649	-1.847
0.10	-5.878	-6.748	-1.534	-1.656
0.15	-4.542	-5.246	-1.453	-1.532
0.20	-4.108	-4.849	-1.229	-1.386
0.25	-3.946	-4.106	-1.116	-1.224
0.30	-3.649	-3.864	-1.058	-1.028
0.35	-3.267	-3.662	-0.986	-0.849
0.40	-2.876	-2.975	-0.742	-0.697
0.45	-2.642	-2.843	-0.523	-0.432
0.50	-2.248	-2.654	-0.369	-0.358
0.55	-2.105	-2.408	-0.225	-0.267
0.60	-1.986	-2.109	-0.108	-0.185

Table T4: An evaluated result of critical temperature T[MeV] of three-body bound state as a function of density na^3 for dimmers, trimmers, BEC and BCS phases

na^3	<-------- T [MeV]---------------→			
	dimmers	Trimmers	BEC	BCS
0.00	0.000	-----	0.000	-----
0.10	0.582	-----	0.205	-----
0.20	0.975	-----	0.308	-----
0.30	1.267	-----	0.492	-----
0.40	2.346	-----	0.786	-----
0.50	4.587	-----	1.126	-----
0.60	6.235	-----	1.167	-----
0.70	----	7.586	1.198	-----
0.80	----	8.243	1.247	-----
0.90	----	9.107	-----	1.297
1.00	----	9.945	-----	1.328
1.10	-----	10.248	-----	1.532
1.20	-----	11.324	-----	1.587
1.30	-----	11.876	-----	1.675
1.40	-----	12.232	-----	1.886
1.50	-----	12.467	-----	1.943

Table T5: An evaluated result of condensate fraction n_0/n of fermionic atoms as a function of the inverse interaction strength $1/k_Fa$, Mean-field results were compared with fixed mode diffusion Monte Carlo results, k_F is Fermi wave number, n is the total number of atoms and a is the s-wave scattering length of the inter-atomic potential

$1/k_Fa$	<------ n_0/n------------------→	
	Mean-field result	Monte Carlo result
-3.00	0.027	-----
-2.50	0.038	-----
-2.00	0.075	-----
-1.50	0.102	-----
-1.00	0.209	0.076
-0.50	0.353	0.225
0.00	0.578	0.376
0.50	0.623	0.583
1.00	0.759	0.647
1.50	0.872	0.759
2.00	1.027	0.875
2.50	1.132	0.923
3.00	1.253	1.103

Table T6: An evaluated results of condensate fraction n_0/n of neutron pairs in neutron matter as a function of the scaled neutron density n/n_S, n_S is nuclear saturation density and n_S =0.16fm^{-3}. The results were obtained by solving equation 28(f) and 28 (I) (Result I) and 28(h) and 28 (I) (Result II) respectively.

n/n_S	< -------- n_0/n ----------------→	
	I Result	II result
10^{-12}	0.000	0.000
10^{-11}	0.000	0.000
10^{-10}	0.000	0.000
10^{-9}	0.052	0.047
10^{-8}	0.078	0.069
10^{-7}	0.092	0.084
10^{-6}	0.107	0.099
10^{-5}	0.125	0.116
10^{-4}	0.167	0.234
10^{-3}	0.224	0.356
10^{-2}	0.323	0.389
10^{-1}	0.456	0.463
10	0.487	0.525
10^{1}	0.408	0.489
10^{2}	0.347	0.392
10^{3}	0.232	0.287

Table T7: An evaluated result for scaled density (n/n_S) as a function of (r/R), Here n_S is nuclear saturation density =0.16fm^{-3} and R is the radius of star. The result is performed for 1.4 solar mass neutron star

r/R	<------ n/n_S-------------→	
	Result obtained by PAL model	Result obtained by Walaca model
0.50	10.276	10.357
0.l70	9.842	9.958
0.75	9.327	8.732
0.80	8.539	7.968
0.85	7.876	7.542
0.90	6.557	6.358
0.95	0.0943	0.0894
1.00	0.0538	0.0476
1.05	0.0476	0.0239
1.10	0.0017	0.0025
1.15	0.0038	0.0017
1.20	0.0059	0.0011
1.25	0.0087	0.0002
1.30	0.009	0.0005
1.40	0.0005	0.0006
1.50	0.0003	0.0007

Table T8: An evaluated result of condensate fraction (n_0/n) of neutron pairs as a function of scaled distance r/R for 1.4 solar mass neutron star. Result I is obtained by solving equation 28 (f) and 28 (I) and (result II) is obtained by solving 28 (h) and 28 (I) respectively

r/R	< ------- (n_0/n)-----------------→	
	Result I	Result II
0.65	0.000	0.000
0.70	0.000	0.000
0.75	0.000	0.000
0.80	0.000	0.000
0.85	0.054	0.067
0.90	0.087	0.095
0.95	0.259	0.387
1.00	0.478	0.505
1.05	0.322	0.452
1.10	0.279	0.316
1.15	0.184	0.255
1.20	0.123	0.158
1.25	0.097	0.117
1.30	0.084	0.095
1.40	0.062	0.072
1.50	0.007	0.009

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