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A PATH FOR HORIZING YOUR INNOVATIVE WORK

OPERATION CALCULUS OF LAPLACE-WEIERSTRASS TRANSFORM

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Abstract: - In this paper we have obtain Laplace-Weierstrass (LW) transform of some common functions. We discuss disambiguation of Laplace-Weierstrass transform which measures the sensitivity to change of one quantity to another quantity which gives formulae for $LW\{f^{'}(t,y)\}$ and $LW\{f^{*}(t,y)\}$, where $f^{'}$ is the differentiation of f with respect to $'t'$ and f^{*} is the differentiation of f with respect to $'y'$. The technique involve here is integral and differential operators. Also we have given $LW\{f^{'}(t,y)\}$ and $LW\{f^{*}(t,y)\}$ of some common functions by using above formulae and presented them in tabular form.

Keywords: Functions, differential operator, Laplace-Weierstrass transform.

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INTRODUCTION

The Laplace-Weierstrass transform is a technique for solving differential equations. Here differential equation of time domain form is first transformed to algebraic equation of frequency domain form. After solving the algebraic equation in frequency domain, the result then is finally transformed to time domain form to achieve the ultimate solution of the differential equation.

Mathurkar and Gulhane [1, 2, 3] studied Laplace transform associated with Weierstrass transform called LW transform, which is defined as $F(s,x) = LW\{f(t,y)\} = \frac{1}{\sqrt{4\pi}} \int_0^\infty \int_0^\infty f(t,y)\, e^{-st-\frac{(x-y)^2}{4}}\, dy\, dt$ and also studied elementary properties with analytic behaviour. Mathurkar and pawar [4] proves modulation theorem for Laplace-Weierstrass transform. Pathak [5] extended integral transform to the compact support. Widder [6, 7] discussed Laplace as well as Weierstrass transform. Wolf [8] created integral transform.

The paper is organized as follows. Section [II] gives the Laplace-Weierstrass transform of some common functions presented in tabular form. In section [III] Disambiguation theorem is discussed giving formulae for $LW\{f^{'}(t,y)\}$ and $LW\{f^{*}(t,y)\}$. In Section [IV] $LW\{f^{'}(t,y)\}$ and $LW\{f^{*}(t,y)\}$ of some common functions is presented in tabular form. Lastly the conclusion is stated.

II. Laplace-Weierstrass transform of some common functions

Table 1:

Sr. No.	Functions	LW transform
1.	e^{at+by}	$\frac{-1}{2(s-a)} e^{bx+b^2} erf_c\left(\frac{x}{2}+b\right)$
2.	1	$\frac{-1}{2s} erf_c\left(\frac{x}{2}\right)$

3.	$\sin(at+by)$	$\frac{1}{4i}\left[\frac{1}{s+ia}e^{-ibx-b^2}erf_c\left(\frac{x}{2}-ib\right)-\frac{1}{s-ia}e^{ibx-b^2}erf_c\left(\frac{x}{2}+ib\right)\right]$
4.	$\cos(at+by)$	$-\frac{1}{4}\left[\frac{1}{s+ia}e^{-ibx-b^2}erf_c\left(\frac{x}{2}-ib\right)+\frac{1}{s-ia}e^{ibx-b^2}erf_c\left(\frac{x}{2}+ib\right)\right]$
5.	$\sinh(at+by)$	$\frac{1}{4}\left[\frac{1}{s+a}e^{-bx+b^2}erf_c\left(\frac{x}{2}-b\right)-\frac{1}{s-a}e^{bx+b^2}erf_c\left(\frac{x}{2}+b\right)\right]$
6.	$\cosh(at+by)$	$-\frac{1}{4}\left[\frac{1}{s+a}e^{-bx+b^2}erf_c\left(\frac{x}{2}-b\right)+\frac{1}{s-a}e^{bx+b^2}erf_c\left(\frac{x}{2}+b\right)\right]$
7.	$Sin\,at\ Sin\,by$	$\frac{s}{4(s^2+a^2)}\left\{e^{-ibx-b^2}\,erf_c\left(\frac{x}{2}-ib\right)+e^{ibx-b^2}erf_c\left(\frac{x}{2}+ib\right)\right\}$
8.	$Cos\,at\ Cos\,by$	$\frac{ia}{4(s^2+a^2)}\left\{e^{-ibx-b^2}\,erf_c\left(\frac{x}{2}-ib\right)-e^{ibx-b^2}erf_c\left(\frac{x}{2}+ib\right)\right\}$
9.	$Sin\,at\ Cos\,by$	$\frac{-a}{4(s^2+a^2)}\left\{e^{-ibx-b^2}\,erf_c\left(\frac{x}{2}-ib\right)-e^{ibx-b^2}erf_c\left(\frac{x}{2}+ib\right)\right\}$
10.	$Cos\,at\ Sin\,by$	$\frac{s}{4i(s^2+a^2)}\left\{e^{-ibx-b^2}\,erf_c\left(\frac{x}{2}-ib\right)+e^{ibx-b^2}erf_c\left(\frac{x}{2}+ib\right)\right\}$

III. Derivative Theorem:

Let $f(t,y)$ be continuous for all $(t,y)\geq 0$ and be of exponential order as $t,y\rightarrow\infty$ and $f'(t,y)$ $and\ f^{*}(t,y) are$ of class A, then Laplace-Weierstrass transform of the derivatives

$f'(t,y)\ and\ f^{*}(t,y)$ Exists. We have obtained the following formulae for $LW\{f'(t,y)\}\ \&\ LW\{f^{*}(t,y)\}$

$$LW\{f'(t,y)\}=-\frac{1}{\sqrt{4\pi}}\int_0^\infty e^{\frac{-(x-y)^2}{4}}f(0,y)\,dy+s\,LW\{f(t,y)\}$$

Where $f^{'}$ is the differentiation of f with respect to $'t'$.

$$LW\left\{ f^{*}(t,y)\right\} = -\frac{1}{\sqrt{4\pi}}\int_0^\infty e^{-st-\frac{x^2}{4}} f(t,0)\, dt - \frac{x}{2} LW\{ f(t,y)\} + \frac{1}{2} LW\{y\, f(t,y)\}$$

Where f^{*} is the differentiation of f with respect to $'y'$.

Proof: Here given that the functions $f^{'}(t,y)\; and\; f^{*}(t,y)\; are$ piecewise continuous on every finite interval in the range $(t,y)\geq 0$.

$\Rightarrow e^{-st-\frac{(x-y)^2}{4}} f^{'}(t,y)\; and\; e^{-st-\frac{(x-y)^2}{4}} f^{*}(t,y)\; are$ R-integrable over any finite interval in the range $(t,y)\geq 0$.

Now by definition of LW transform, $LW\{f^{'}(t,y)\}$ is given by,

$$LW\left\{ f^{'}(t,y)\right\} = \frac{1}{\sqrt{4\pi}}\int_0^\infty\int_0^\infty e^{-st-\frac{(x-y)^2}{4}} f^{'}(t,y)\,dt\,dy$$

$$= \frac{1}{\sqrt{4\pi}}\int_0^\infty e^{\frac{-(x-y)^2}{4}} \left\{ \left[e^{-st} f(t,y)\right]_0^\infty - \int_0^\infty e^{-st}(-s)\, f(t,y)\,dt \right\} dy$$

$$= -\frac{1}{\sqrt{4\pi}}\int_0^\infty e^{\frac{-(x-y)^2}{4}} f(0,y)\, dy + s\, LW\{ f(t,y)\} \qquad (III.1)$$

And

$LW\{f^{*}(t,y)\}$ is given by,

$$LW\left\{ f^{*}(t,y)\right\} = \frac{1}{\sqrt{4\pi}}\int_0^\infty\int_0^\infty e^{-st-\frac{(x-y)^2}{4}} f^{*}(t,y)\,dy\,dt$$

$$= \frac{1}{\sqrt{4\pi}}\int_0^\infty e^{-st} \left\{ \left[e^{\frac{-(x-y)^2}{4}} f(t,y)\right]_0^\infty - \frac{1}{2}\int_0^\infty e^{\frac{-(x-y)^2}{4}} (x-y)\, f(t,y)\,dy \right\} dt$$

$$= -\frac{1}{\sqrt{4\pi}}\int_0^\infty e^{-st-\frac{x^2}{4}} f(t,0)\, dt - \frac{x}{2} LW\{ f(t,y)\} + \frac{1}{2} LW\{y\, f(t,y)\} \qquad (III.2)$$

IV. We have obtained *LW{ f ′(t, y)}* & *LW{ f *(t, y)}* of some common functions and presented them in the form of following tables

Table 2:

Sr. No.	Functions	$LW\left\{ f'(t,y)\right\}$ transform
1.	e^{at+by}	$-\dfrac{a\,e^{bx+b^2}}{2(s-a)}erf_c\left(\dfrac{x}{2}+b\right)$
2.	*1*	0
3.	*sin(at+by)*	$\dfrac{-a}{4(s-ia)}e^{ibx-b^2}erf_c\left(\dfrac{x}{2}+ib\right)-\dfrac{a}{4(s+ia)}e^{-ibx-b^2}erf_c\left(\dfrac{x}{2}-ib\right)$
4.	*cos(at+by)*	$\dfrac{-ia}{4(s-ia)}e^{ibx-b^2}erf_c\left(\dfrac{x}{2}+ib\right)+\dfrac{ia}{4(s+ia)}e^{-ibx-b^2}erf_c\left(\dfrac{x}{2}-ib\right)$
5.	*sinh(at+by)*	$\dfrac{-a}{4(s-a)}e^{bx+b^2}erf_c\left(\dfrac{x}{2}+b\right)-\dfrac{a}{4(s+a)}e^{-bx+b^2}erf_c\left(\dfrac{x}{2}-b\right)$
6.	$\cosh(at+by)$	$\dfrac{-a}{4(s-a)}e^{bx+b^2}erf_c\left(\dfrac{x}{2}+b\right)+\dfrac{a}{4(s+a)}e^{-bx+b^2}erf_c\left(\dfrac{x}{2}-b\right)$

Table 3:

Sr. No.	Functions	$LW\left\{ f^*(t,y)\right\}$ transform
1.	e^{at+by}	$\dfrac{-b}{2}\,\dfrac{e^{bx+b^2}}{s-a}erf_c\left(\dfrac{x}{2}+b\right)$
2.	*1*	0
3.	*sin(at+by)*	$\dfrac{-b}{4(s-ia)}e^{ibx-b^2}erf_c\left(\dfrac{x}{2}+ib\right)-\dfrac{b}{4(s+ia)}e^{-ibx-b^2}erf_c\left(\dfrac{x}{2}-ib\right)$

4.	*cos(at+by)*	$\frac{-ib}{4(s-ia)}e^{ibx-b^2}erf_c\left(\frac{x}{2}+ib\right)+\frac{ib}{4(s+ia)}e^{-ibx-b^2}erf_c\left(\frac{x}{2}-ib\right)$
5.	*sinh(at+by)*	$\frac{-b}{4(s-a)}e^{bx+b^2}erf_c\left(\frac{x}{2}+b\right)-\frac{b}{4(s+a)}e^{-bx+b^2}erf_c\left(\frac{x}{2}-b\right)$
6.	$\cosh(at+by)$	$\frac{-b}{4(s-a)}e^{bx+b^2}erf_c\left(\frac{x}{2}+b\right)+\frac{b}{4(s+a)}e^{-bx+b^2}erf_c\left(\frac{x}{2}-b\right)$

V. Conclusion:

In this paper we have seen that how we obtained the derivative of Laplace-Weierstrass transform and also the results of various functions. In this paper a technique involving integral and differential operators has been used to affect the transform. Since this transform is an important tool in signal processing and many other branches of engineering, it provides new aspects to many mathematical disciplines such as transform theory, functional analysis, differential equation etc.

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