Research Article CODEN: IJPAKY Impact Factor: 4.226 ISSN: 2319-507X
Ashok Kumar Singh, IJPRET, 2018; Volume 6 (6): 51-72 IJPRET

INTERNATIONAL JOURNAL OF PURE AND APPLIED RESEARCH IN ENGINEERING AND TECHNOLOGY

A PATH FOR HORIZING YOUR INNOVATIVE WORK

A THEORETICAL STUDY OF TWO-FERMION BOUND STATE IN A BOSE-EINSTEIN CONDENSATE AND EVALUATION OF MOLECULAR CONDENSATE FRACTION N_0/N AS A FUNCTION OF SCALED TEMPERATURE T/T_C

ASHOK KUMAR SINGH[1] AND L. K. MISHRA[2]

1. Department of Physics, IDBPS College, Garh Nokha, Rohtas-802215(Bihar)
2. Department of Physics, Magadh University, Bodh-Gaya-824234 (Bihar)

Accepted Date: 16/01/2018; Published Date: 01/02/2018

Abstract: - Using the theoretical formalism of **W. Zhang etal [arXiv:cond-mat/0206249v1[cond.mat stat.mech]]13 Jun 2002**,we have studied two fermion bound state in a Bose-Einstein condensate. Our theoretically obtained results show that the condensate phonon mode can provide a nonlinear medium for the fermions atoms. Using the theoretical formalism of **K. M. O' Hara etal[Phys. Rev. A66**, **041401(R) (2002)],** we have studied a zero-crossing in the scattering length of a mixture of the two lowest hyperfine states of ^{6}Li. The crossing is obtained near magnetic field of 65mT. This is obtained by the help of Feshbach resonance. Using the theoretical formalism of **Markes Greiner etal {Nature, 26 Nov (2005)],** we have studied the emergence of molecular Bose-Einstein condensate from a Fermi gas. We have also observed the BCS-BEC crossover situation. The BEC appears on the repulsive side of the Feshbach resonance and BCS-type on the attractive side of the resonance. Our theoretically obtained results are in good agreement with those of other theoretical workers.

Keywords: Nonlinear Schrodinger equation, Healing length of the condensate, Effective Kerr medium of the fermion field, Zero-crossing of the scattering length, BCS-BEC crossover.

Corresponding Author: MR. ASHOK KUMAR SINGH

Access Online On:

www.ijpret.com

How to Cite This Article:

Ashok Kumar Singh, IJPRET, 2018; Volume 6 (6): 51-72

PAPER-QR CODE

INTRODUCTION

A decade of experiments with degenerate fermionic quantum gases has delivered major scientific advances as a whole new class of quantum many-body systems[1-3]. Feshbach resonances[4] played a crucial role in this development because it offers exceptional control over the interatomic interactions at low temperatures[5]. In gases with appropriate spin mixture, the sign and magnitude of s-wave scattering length ‘a’ can be tuned to any positive and negative value by choosing the proper magnetic field in the vicinity of resonances. In the case of fermionic atoms, the role of Feshbach resonance is especially remarkable because Pauli exclusion principle dramatically suppresses three-body loses to deeply bound molecular states[6,7]. The tenability has been used with great success in two component Fermi gases of ^{6}Li and of ^{40}K to study and control pairing mechanism. This pairing mechanism includes both of the Cooper type on the attractive side of the resonance[8] ($a<0$) and of repulsive side ($a>0$). In particular, the universal crossover from super fluidity of a molecular Bose-Einstein condensate (BEC) towards the Bardeen Cooper Schrieffer (BCS) limit has received a lot of attention[9,10]. Essential for these studies is the availability of sufficiently broad Feshbach resonance in the ^{6}Li and ^{40}K homonuclear gas. These studies have strongly gained interest due to additional mass imbalance. Theoretical studies of these mixtures include super fluidity[11], phase transitions[12], crystalline phases[13], exotic pairing mechanism[14] and long lived timers[15]. Many of these studies require the mixture to be strongly interacting. In the universal limit i.e the scattering length should be very large and the only parameter that determines the two-body interactions is the scattering length. The first mass-imblanced ultra cold fermionic mixture has been realized namely, a mixture of the only stable fermionic alkaline species[16] ^{6}Li and ^{40}K. The basic interaction properties of $^{6}Li/^{40}K$ system were established in experiments[17,18].

In this paper, using the theoretical formalism of W. Zhang etal.[19], we have studied two-fermion bound state in a Bose-Einstein condensate. We observed that that the condensate phonon mode can provide a nonlinear medium for the formation of two-fermion bound sate. Using the theoretical formalism of K. M. O’Hara etal.[20], we have studied a zero-crossing in the scattering length of a mixture of two lowest hyperfine sates of ^{6}Li atoms. This can be achieved with the help of Feshbach resonance. The crossing is obtained near a magnetic field of 65mT. Using the theoretical formalism of Markes Greiner etal.[21], we have studied the emergence of molecular Bose-Einstein condensate from a Fermi gas. We observed The BCS-BEC cross over Physics by evaluating condensate fraction N_0/N as a function of scaled temperature T/T_F. Our obtained results are in good agreement with other theoretical workers[22,23].

MATERIALA AND METHODS

One studies two-fermion bound state in a Bose-Einstein condensate (BEC) in a situation, where a gas of bosons serves as a nonlinear medium for fermionic atoms. In particular, one studies how the inter atomic interactions between a Bose-Einstein condensate and a fermionic beam can be employed to manipulate the quantum state of the beam. By drawing on the analogy to nonlinear optics, one describes the interaction in terms of an effective Kerr nonlinearity. One shows that a two-fermionic bound state can result with a unique signature in nonlinear atom optical experiment. Since the Pauli exclusion principle precludes the direct evaporative cooling of spin –polarized fermionic samples, current experiment employ either unpolarized fermionic mixtures[24-26] or Bose-Fermi mixtures[27]. The starting point is a beam of fermionic atoms with two internal spin states $|\uparrow\rangle$ and $\langle\downarrow|$ interacting via two-body collisions with a quantum degenerate Bose gas.

The atomic Bose field is decomposed in the following way

$$\psi_B(r^{\rightarrow}) = \phi_B(r^{\rightarrow}) + \xi(r^{\rightarrow}) = \sqrt{n_B(r^{\rightarrow})} + \xi(r^{\rightarrow}) \quad (1)$$

Where $\phi_B(r^{\rightarrow})$ is the condensate wave function taken to be real for simplicity, $\xi(r^{\rightarrow})$ describes the elementary excitations above the condensate and $n_B = |\phi_B|^2$ is the condensate density. The fermionic field is described by the field operators $\psi_\sigma(r^{\rightarrow})$ with $\sigma = \{\uparrow, \downarrow\}$. One introduces desity fluctuations of the fermionic fields as

$$\delta n_\sigma = \psi_\sigma{}^{\dagger}(r^{\rightarrow})\psi_\sigma(r^{\rightarrow}) - \left\langle \psi_\sigma{}^{\dagger}(r^{\rightarrow})\psi_\sigma(r^{\rightarrow}) \right\rangle \quad (2)$$

The Bose and Fermi systems are coupled for two-body interactions. Now in the simplest approximation and taking into account that s-wave scattering is forbidden fermionic atoms of the same spin, one finds that to lowest order in $\xi(r^{\rightarrow})$, the dynamics of the bosonic atoms is given by the coupled equations[28]

$$[H^{(0)}{}_B + g n_B(r^{\rightarrow}) + \sum_\sigma f_\sigma n_\sigma(r^{\rightarrow})]\phi_B = \mu_B \phi_B \quad (3)$$

$$i\hbar\frac{\partial\xi}{\partial t}=(H_B-\mu_B)\xi+g\phi^2\xi^\dagger+\phi\sum_\sigma f_\sigma\delta n_\sigma \quad (4)$$

In the same way, the fermionic field equations are

$$i\hbar\frac{\partial\psi_\sigma}{\partial t}=(H_{F\sigma}-\mu_\sigma)\psi_\sigma + f_\sigma(\phi_B\xi^\dagger+\phi^*{}_B\xi)\psi_\sigma+h\psi_{\sigma'}{}^\dagger\psi_{\sigma'}\psi_\sigma \quad (5)$$

Here, $$H^{(0)}{}_\alpha=T_\alpha+V_\alpha(\alpha=B,\sigma) \quad (6)$$

This is the single-particle Hamiltonians for bosonic atoms and for fermionic atoms of spin σ respectively. T_α and V_α are the associated kinetic energy and trapping potential. The Hamiltonians

$$H_B=H^{(0)}{}_B+2gn_B+\Sigma_\sigma f_\sigma n_\sigma \quad (7)$$

$$H_{F\sigma}=H^{(0)}{}_{F\sigma}+f_\sigma n_B \quad (8)$$

These Hamiltonians also include the self-contribution to the mean field energy of the respective fields. μ_α are the chemical potential. The parameters g, f and h represent the boson-boson, boson-fermion and fermion-fermions interaction strengths.

$$g=\frac{4\pi\hbar^2a_B}{m_B} \quad 9(a)$$

$$f_\sigma=\frac{2\pi\hbar^2a_{BF\sigma}}{m_r} \quad 9(b)$$

$$h=\frac{4\pi\hbar^2a_F}{m_F} \quad 9(c)$$

$$m_r=\frac{m_Bm_F}{(m_B+m_F)} \quad 9(d)$$

Here, α is s-wave scattering length and m_r is the reduced mass. Now, one is interested to study how the presence of a condensate can induce nonlinear dynamics of a fermion field. One considers a situation where the back-action of the fermionic fields on the condensate is negligible. One assumes that $gn_B \gg \Sigma_\sigma f_\sigma n_\sigma$ is satisfied. In this case, one ignors the effects of fermionic beam on the condensate wave function $\phi_B(\vec{r})$. One then applies a standard Bogoliubov approach[29] to determine the effect of fermions on the excitation field $\xi(\vec{r})$. One then finds here

$$\xi(\vec{r},t) = \xi^{(0)}(\vec{r},t) + \frac{1}{i\hbar}\int_0^t dt'[G(\vec{r},\vec{r}',t-t')\phi_B(\vec{r}') - F(\vec{r},\vec{r}',t-t')\phi^*(\vec{r})]\Sigma_\sigma f_\sigma \delta n_\sigma \quad (10)$$

The term $\xi^{(0)}(\vec{r}t)$ on the right hand side of the above equation describes the free-field quasi-particle fluctuations in the absence of fermions. It has the familiar form

$$\xi^{(0)}(\vec{r}t) = \Sigma_n[u_n(\vec{r})e^{-iE_n t/\hbar}\alpha_n - v^*_n(\vec{r})e^{iE_n t/\hbar}\alpha^\dagger_n] \quad (11)$$

Here the Bogoliubov quasi-particle operators α_n and $\alpha^\dagger_n$ satisfy Bose commutation relations. The quasi-particle mode functions $u_n(\vec{r})$ and $v_n(\vec{r})$ and corresponding eigenvalues are determined by the matrix equations

$$\begin{pmatrix} H_B & -gn_B \\ gn_B & -H_B \end{pmatrix}\begin{pmatrix} u_n(\vec{r}) \\ v_n(\vec{r}) \end{pmatrix} = E_n\begin{pmatrix} u_n(\vec{r}) \\ v_n(\vec{r}) \end{pmatrix} \quad (12)$$

The second term of equation (10) is a four-wave mixing process that mixes the condensate with quasi-particles. It is mediated by the density fluctuations of the Fermi fields. The evaluation is governed by the quasi-particle Green's function

$$G(\vec{r},\vec{r}',\tau) = \Sigma_n[e^{-iE_n\tau/\hbar}u_n(\vec{r})u^*_n(\vec{r}') - e^{iE_n\tau/\hbar}v^*_n(\vec{r})v_n(\vec{r}')] \quad (13)$$

There is similar form for $F(\vec{r},\vec{r}',\tau)$ but with $u^*_n(\vec{r}')$ and $v_n(\vec{r}')$ are replaced by $v^*_n(\vec{r}')$ and $u_n(\vec{r}')$ respectively.

The lowest –order contributions of the condensate to the dynamics of the Fermi fields is obtained by substituting equation (10) in to equation (5). Using the homogeneous case quasi-particle mode functions for simplicity, this yields the Heisenberg equations of motion

$$i\hbar\frac{\partial\psi_\sigma}{\partial t}=(H_{F\sigma}-\mu_\sigma)\psi_\sigma+\hbar\psi^\dagger{}_{\sigma'}\psi_{\sigma'}\psi_\sigma+\Sigma_{\sigma'=\uparrow,\downarrow}$$

$$\int d^3\tau'\int_0^t d\tau W_{\sigma\sigma'}(\vec{r},\vec{r}',\tau)$$

$$\phi_B(\vec{r})\phi_B(\vec{r}')\delta n_{\sigma'}(\vec{r}',t-\tau)\psi_\sigma+\Gamma_\sigma\psi_{\sigma'} \qquad (14)$$

Here we have defined

$$W_{\sigma\sigma'}(\vec{r},\vec{r}',\tau)=(\frac{1}{i\hbar})[\Delta_{\sigma\sigma'}(\vec{r},\vec{r}',\tau)-\Delta^*{}_{\sigma\sigma'}(\vec{r},\vec{r}',\tau)] \qquad 15(a)$$

$$\Delta_{\sigma\sigma'}=\frac{f_\sigma f_{\sigma'}}{V}\Sigma_k\sqrt{\frac{\varepsilon_k}{\varepsilon_k+2gn_B}}e^{-iE_k\tau/\hbar+ik.(\vec{r}-\vec{r}')} \qquad 15(b)$$

$$\varepsilon_k=\frac{\hbar^2k^2}{2m_B} \qquad 15(c)$$

$$E_K=\sqrt{\varepsilon_k(\varepsilon_k+2gn_B)} \qquad 15(d)$$

$$\Gamma_\sigma=f_\sigma\phi_B(\vec{r})[\xi^{(0)}(\vec{r},t)+h.c] \qquad 15(e)$$

$$\langle\Gamma_\sigma(\vec{r},t)\Gamma_{\sigma'}(\vec{r}',t')\rangle=n_B\Delta_{\sigma\sigma'}(\vec{r},\vec{r}',t-t') \qquad 15(f)$$

The third term in equation (14) is the nonlinear optics analogue of the fifth-order nonlinearity involving two condensate fields, the fermion density fluctuations which are quadratic in the fermion fields and the fermion fields. These five fields can mix to produce a sixth, the physical process involved a sixth –wave mixing between the boson and fermion fields. The term shown by equation 15(e) is a stochastic potential whose physical origin are the density fluctuations of the vacuum state of the Bose quasi-particles. The two-point correlation function is shown in equation 15(f)

Now, one assumes that the fermion beam propagates through the condensate at velocity v less than the condensate sound velocity c= $=\frac{\hbar\sqrt{4\pi n_B a_B}}{m_B}$. Viewing the beam as an impurity traversing the condensate, the condition v<c implies that it will not create incoherent phonon excitations that persist in the condensate after the beam has passed[30]. Physically, in this limit the fermion beam is accompanied as it propagates by a virtual phonon cloud that mediates an effective instantaneous interaction between the fermions. Furthermore, since the phonon excitations are virtual the effects of the stochastic potential may be safely neglected. One may neglect time retardation in the collision term and the stochastic potential in the fermionic field (14). It then reduces to

$$i\hbar\frac{\partial\psi_\sigma(\vec{r},t)}{\partial t}=(H_{F\sigma}-\mu_\sigma)\psi_\sigma+\hbar\psi^\dagger{}_{\sigma'}\psi_{\sigma'}\psi_\sigma$$
$$+\Sigma_{\sigma'=\uparrow,\downarrow}\int d^3r' U_{\sigma\sigma'}(\vec{r},\vec{r}')\delta n_{\sigma'}(\vec{r}',t)\,\psi_\sigma(\vec{r}',t) \quad (16)$$

where for the case of a homogeneous condensate $U_{\sigma\sigma'}(\vec{r},\vec{r}')$ is the Yukawa potential[31,32]

$$U_{\sigma\sigma'}(\vec{r},\vec{r}')=-\left(\frac{m_B f_\sigma f_{\sigma'} n_B}{\pi\hbar^2}\right)\frac{e^{-\sqrt{2}|\vec{r}-\vec{r}'|/l_h}}{|\vec{r}-\vec{r}'|} \quad (17)$$

$l_h=\hbar/\sqrt{2m_B g n_B}$ is the healing length of the Bose condensate. The negative sign in equation (17) indicates that Fermi-Bose coupling induces an attractive effective force between fermionic atoms. In the language of nonlinear optics the condensate acts as nonlinear crystal for the fermions. The induced effective fermion-fermion interaction $U_{\sigma\sigma'}(\vec{r},\vec{r}',t)$ plays the role of a spatially non-local Kerr nonlinearity for the fermion field. Here, one consider the case of formation of two-fermion bound , a problem which is closely related to the quantum propagation of a two-photon light beam in a self-focusing medium[33].

One assumes that an ultra cold source of fermionic atoms generates a beam containing simultaneously two atoms. One considers a spin-polarized beam described by two-atom quantum state.

$$|\phi(t)\rangle = \frac{1}{\sqrt{2}}\int d^3r_1\int d^3r_2 f(\vec{r}_1,\vec{r}_2,t)\psi^\dagger(r_1)\psi(r_2)|0\rangle \quad (18)$$

This state propagates at velocity v into a condensate. The function $f(\vec{r}_1,\vec{r}_2,t)$ gives the probability amplitude to find one atom at **r₁** and other at **r₂** and is antisymmetric under atomic exchange. The time evaluation of $|\phi(t)\rangle$ is determined by the Schrodinger equation

$$i\hbar\frac{\partial|\phi\rangle}{\partial t} = H_{eff}|\phi\rangle \quad (19)$$

$$H_{eff} = \int d^3r\psi^\dagger(\vec{r})[H_F - \mu_F)\psi(\vec{r}) + \frac{1}{2}\iint d^3r d^3r' U(\vec{r},\vec{r}')\psi^\dagger(\vec{r})\delta n\left(\vec{r}'\right)\psi(\vec{r}) \quad (20)$$

This yields the equation of motion for $f(\vec{r}_1,\vec{r}_2,t)$ which is given by

$$i\hbar\frac{\partial f(\vec{r}_1,\vec{r}_2,t)}{\partial t} = (H_F(\vec{r}_1) + H_F(\vec{r}_2) + U(\vec{r}_1,\vec{r}_2) + 2\iint d^3r' d^3r''[U(\vec{r}_1,\vec{r}') + U(\vec{r}_2,\vec{r}'')]|f(\vec{r}',\vec{r}'',t)|^2)f(\vec{r}_1,\vec{r}_2,t) \quad (21)$$

This clearly shows the role of the effective Kerr medium on the fermionic field. To determine the condition for the formation of the two-atom bound state of equation (21) one transform into the center of mass coordinates $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$. One also make the ansatz as $f(\vec{r}_1, \vec{r}_2, t) = e^{i\vec{K}.\vec{R} - iEt/\hbar} W(\vec{r})$ This leads to finally an eigen value problem for the relative motion of two particles in a Yukawa potential

$$\left(-\frac{\hbar^2 \nabla^2_r}{2\mu} - \frac{U_0}{r} e^{-\sqrt{2}r/l_h}\right) W(\vec{r}) = E_r W(\vec{r}) \qquad (22)$$

Here, $U_0 = f^2 m_B n_B / (\pi\hbar^2)$ is the potential strength, $\mu = m_F / 2$ is the reduced mass, and E_r is the energy of relative motion. From equation (22), one can see that the spatial width of the energy eigenstates is determined by the length scale $l_0 = \hbar^2 / (\mu U_0)$. The energy eigenvalues of a Yukawa potential have been investigated. An accurate formulae[34] has been provided for the number of bound states for a state with total angular momentum quantum number l

$$\nu = (\sqrt{Z} - \sqrt{Z_l}) S_l + 1 \qquad (23)$$

where $Z = l_h / (\sqrt{2} l_0)$, $Z_l = 0.8399(1 + 2.7359l + 1.6242l^2)$ and $S_l = 1.1335(1 + 0.0191l - 0.001684l^2)$. Now considering the requirements of symmetry of spatial wave function for the spin polarized beam, the angular momentum must be l=1, 3, 5,------.The condition that there exists at least one bound state is $Z \geq Z_1$. This determines a spatial range $l_0 \leq l_b = l_h / 6.366$ for a two atom bound state in a Bose condensate which enforces a requirement of sufficient potential strength $U_0 \geq U_b = 6.366 / (\mu l_h)$ for binding.

As is well known, a Feshbach resonance arises in a colliding two-atom system when the energy of the incoming open elastic channel is magnetically tuned into resonance with a bound molecular state of an energetically closed channel. The tuning dependence arises from the difference in the magnetic moments of the open and closed channels[35]. In the vicinity of Feahbach resonance, the s-wave scattering length a(B) is described by[36]

$$a(B) = a_{bg}[1 - \frac{\Delta B}{B - B_0}] \quad (24)$$

Where B is applied field, a_{bg} is the background scattering length, B_0 is the field at which the resonance occurs, ΔB is proportional to the strength of the coupling between the open and closed channels.

RESULTS AND DISCUSSION

Using the theoretical formalism of W. Zhang etal.[19], we have theoretically studied the two fermion bound states in a Bose-Einstein condensate. Here, one has derived a nonlinear Schrodinger equation for a fermionic beam in a condensate and shown that the condensate acts as an effective nonlinear medium for the fermions. This leads to a formation of two-atom bound state which is closely analogous to two- photon bound state in a self focussing medium[33]. Using the theoretical formalism of Markus Greiner etal.[21], we have studied the super fluidity in a dilute gas of fermionic atoms and evaluated the molecular condensate fraction (N_0/N) as a function of scaled temperature (T/T_C). In **table T1**, we have shown the average energy per particle $\frac{E}{Nk_\beta T}$ as a function of temperature (T/T_F) for two values of hyperfine state m_f =7/2 and m_f =9/2 from Fermi gas prediction. The expt. values[37] are also shown with them. Our theoretically evaluated results show that Fermi gas prediction has a satisfactory match with the expt. values of m_f =7/2 and 9/2. Here m denotes the magnetic quantum number and f is the total atomic angular momentum quantum number. These parameters give the resonance effect of collisions between the atoms in the lowest energy spin state. In **table T2**, we have presented the evaluated results of molecular condensate fraction (N_0/N) as a function of scaled temperature (T/T_C). Here N is the total number of molecules when there is no change of magnetic field for expansion and T_C is determined by a formulae.

$$T_C = 0.94(N\nu^2{}_F\nu_z)^{\frac{1}{3}}\frac{h}{k_\beta}$$

$$T_F = (6N\nu^2{}_F\nu_z)^{\frac{1}{3}}\frac{h}{k_\beta}$$

Here N is the particle number in each spin state. ν_F and ν_z are axial and radial trap frequencies having values 430 MHz and 250 MHz respectively. Our theoretically evaluated results are in good agreement with the expt. data[38]. In **table T3**, we have shown the expt. result of dependence of condensate fraction (N_0/N) as a function of time(ms) in which the magnetic field is ramped across the Feshbach resonance. This is the dependence of condensate fraction on magnetic field sweep rate and measurement of condensate life time. In **table T4**, we have presented the results of analytic model of the scattering length in the unit of a_0 for a mixture of two lowest hyperfine states of ^6Li as a function of magnetic field (mT). The results show that the scattering length has zero crossing. In **table T5**, we have shown the evaluated results of temperature $T(\mu K)$ near the zero crossing of the magnetic field. Our theoretical results show that temperature increases and attains maximum value and then decreases. In **table T6**, we have shown the evaluated results of number of atoms (trapped population)x 10^3 as a function of magnetic field (mT) near the zero crossing of the scattering. We observe that there is a decrease in the trapped population as the magnetic field is increased. We also observe that there is a rise of the trap population near 65mT and this type of magnetic dependence trap population was observed by Dieckmann etal.[39]. In **table T7**, we have presented the results for dependence of the condensate fraction (N_0/N) of the atom-molecule mixture as a function of scaled temperature (T/T_F). The results show that the BEC is formed when the initial temperature is below $0.17T_F$. In **table T8**, we have shown the evaluated results of total expansion energy per particle for the molecular condensates as a function of the scattering length a (a_0). Our theoretically evaluated results show that the energy (nK) increases with scattering length. These results also indicate that the energy dependence is proportional to scattering length a. This linear dependence of energy suggests that the molecule-molecule scattering length is proportional to the atom-atom scattering[40]. There is some recent calculations[41-50] which reveals the similar behaviour.

CONCLUSION;

_From the above theoretical investigation and analysis, we have come across the following conclusions:

(1)Our theoretically evaluated results show that a nonlinear Schrodinger equation for a fermionic beam in a condensate acts as a nonlinear medium for fermions. This leads to the formation of a two-atom bound state closely analogous to a two-photon bound state in the focusing medium.

(2)Two-fermion bound sate has been formed in the combination of alkali atoms such as ^{6}Li-^{7}Li, ^{6}Li-^{23}Na and ^{40}K-^{39}K. The ^{40}K-^{39}K combination appears to be the most promising due to very large boson-fermion scattering length a_{BF}=52.9nm and very small boson-boson scattering length a_B =0.26nm. A bound fermionic states require a condensate density=10^{13}cm^{3}.

(3)Physically, the understanding of fermionic bound states may be important for the manipulation of quantum statistical properties of fermionic atoms like dynamics of Cooper pairing and formation of quantum solitons in ultra cold fermionic atoms.

(4)We have also studied the zero-crossing in a Feshbach resonance in fermionic atoms. Our theoretical results of trap population as a function of magnetic field confirms this fact that there is a zero-crossing in the elastic scattering length of a mixture of two lowest hyperfine sates of ^{6}Li.

(5) We have studied the emergence of molecular Bose-Einstein condensate from a Fermi gas. We observed that BEC of weakly bound molecules start with a gas of ultra cold fermionic atoms. The molecular BEC appears on the repulsive side of the Feshbach resonance and BCS type of super fluidity on the attractive side of resonance

Table T1: A theoretical results for the evidence for quantum degeneracy effect in trapped Fermi gas where average energy per particle ($(\frac{E}{3K_\beta T})$ is evaluated as a function of $(\frac{T}{T_F})$.

$(\frac{T}{T_F})$	< ------ $(\frac{E}{3K_\beta T})$ ------------------→			
	m_f =7/2		m_f =9/2	
	Fermi gas prediction	Expt	Fermi gas Prediction	Expt

0.10	2.25	2.55	2.32	2.27
0.15	2.15	2.43	2.24	2.12
0.20	2.10	2.00	2.12	2.04
0.25	1.87	1.88	1.92	1.86
0.30	1.68	1.69	1.73	1.65
0.35	1.54	1.58	1.64	1.43
0.40	1.50	1.52	1.55	1.37
0.50	1.32	1.43	1.45	1.29
0.60	1.16	1.20	1.23	1.18
0.80	1.00	1.15	1.17	1.10
1.00	1.00	0.92	1.00	0.95
1.20	1.00	0.83	1.00	0.84
1.40	1.00	0.80	1.00	0.72
1.50	1.00	0.78	1.00	0.67
2.00	1.00	0.75	1.00	0.59

Table T2: An evaluated result of molecular condensate fraction $\frac{N_0}{N}$ as a function of scaled temperature $\frac{T}{T_c}$. N is the total number of molecules when there is no change of magnetic field.

$\frac{T}{T_c}$	$\left(\frac{N_0}{N}\right)_{theory}$	$\left(\frac{N_0}{N}\right)_{expt}$
0.40	0.186	0.205

0.45	0.174	0.188
0.50	0.162	0.165
0.55	0.150	0.158
0.60	0.142	0.140
0.65	0.134	0.137
0.70	0.105	0.112
0.75	0.086	0.095
0.80	0.052	0.065
0.85	0.045	0.053
0.90	0.042	0.045
0.95	0.037	0.040
1.00	0.035	0.038
1.10	0.027	0.030
1.20	0.018	0.021

Table T3: Experimental result of dependence of condensate fraction N_0/N as a function of magnetic field sweep rate .Here fraction of condensed molecule is evaluated as a function of time in which the magnetic field is ramped across the Feshbach resonance

Ramp time (ms)	**< ----- N_0/N----------→**
0.0	0.0
1.0	0.022
2.0	0.045
3.0	0.053

4.0	0.065
5.0	0.087
6.0	0.107
7.0	0.115
8.0	0.127
9.0	0.135
10.0	0.142
12.0	0.155
14.0	0.165
15.0	0.127

Table T4: A theoretical result of the analytical model of the scattering length a of mixture of the two lowest hyperfine states of ^{6}Li as a function of magnetic field

Magnetic field (mT)	Scattering length (Unit of a_0)
0.0	0.00
5.0	10.25
10.0	15.87
20.0	100.6
30.0	250.8
40.0	1543.2
50.0	2000.9

60.0	2500.6
80.0	4000.8
100.0	-4725.0
110.0	-4158.0
120.0	-3947.2
130.0	-3540.3
140.0	-2000.5
150.00	-1965.4

Table T5: An evaluated result of temperature T (μK) as a function of magnetic field (mT) near the zero crossing of the scattering length

Magnetic field (mT)	Temperature T (μK)
20	30.6
30	35.9
40	42.6
45	49.5
50	53.4
55	92.8
60	83.7
65	65.4
70	57.3
75	50.8

80	48.6
85	43.2
90	40.8
100	37.5

Table T6: An evaluated result of number of atoms (trapped population)x10^3 as a function of magnetic field (mT) near the zero crossing of the scattering length

Magnetic field (mT)	Trapped population x10^3
20	534.8
30	523,6
40	500.4
45	489.0
500	480.8
55	677.6
60	603.2
65	512.4
70	500.5
75	487.8
80	455.5
85	417.8
90	300.2
95	295.5

100	210.9

Table T7: An evaluated result of dependence of condensate fraction N_0/N of the atom-molecule mixture as a function of T/T_F

T/T_F	N_0/N
0.05	0.165
0.06	0.162
0.08	0.158
0.10	0.145
0.12	0.137
0.14	0.128
0.15	0.110
0.18	0.095
0.20	0.072
0.22	0.065
0.25	0.058
0.30	0.050
0.32	0.045
0.34	0.042
0.35	0.040
0.38	0.038
0.40	0.036

Table T8: An evaluated result of total expansion energy per particle for the molecular condensate as a function of scattering length a (a_0)

Scattering length a(a_0)	Expansion energy (nK)
10	2.76
50	3.87
100	4.24
200	5.56
400	6.92
500	7.68
800	8.14
1000	9.29
1200	10.28
1400	11.39
1500	12.45
1800	13.86
2000	14.29
2500	15.56
3000	18.27

REFERENCES

1. B. Demarco and D. S. Jin, Science, 285, 1703 (1999)
2. S. Giorgini etal., Rev. Mod. Phys. 80, 1215 (2008)

3. Proceedings of the International school of Physics, Enrico Fermi 'Course CLXIV, edited by M. Ingescio, W. Ketterle and S. Solemon (IOS, Press, Amsterdam, 2008)

4. E. Tiesinga etal., Phys. Rev. A47, 4114 (1993)

5. C. Chin etal., Rev. Mod. Phys. 82, 1225 (2010)

6. C. A. Regal etal., Phys. Rev. Lett. (PRL) 92, 083201 (2004)

7. D. S. Petrov etal., Phys. Rev. Lett. (PRL) 93, 090404 (2004)

8. M. W. Zwierlein etal., Phys. Rev. Lett. (PRL) 92, 120403 (2004)

9. M. Greiner etal., Nature (London) 426, 537 (2003)

10. Y. Ohashi etal. Phys. Rev. Lett. (PRL) 89, 130402 (2002)

11. Y. Ohash and A. Griffen, Phys. Rev. A67, 033603 (2003)

12. I. Bausmerth etal., Phys. Rev. A79, 043622 (2009)

13. D. S. Petrov etal., Phys. Rev. Lett. (PRL) 99, 130407 (2007)

14. M. M. Forbes etal., Phys. Rev. Lett. (PRL) 94, 017001 (2005)

15. J. Levinsen etal., Phys. Rev. Lett. (PRL) 103, 153202 (2009)

16. M. Taglieber etal., Phys. Rev. Lett. (PRL) 100, 010401 (2008)

17. E. Will etal., Phys. Rev. Lett. (PRL) 100, 053201 (2008)

18. A. C. Voigt etal., Phys. Rev. Lett. (PRL) 102, 020405 (2009)

19. W. Zhang etal., arXiv:cond-mat/0206249v1, 13 June (2002)

20. K. M. O'Hara etal., Phys. Rev. A66, 041401(R) (2002)

21. M. Greiner etal, Nature, 26 Nov (2003)

22. Ultra Cold Bosonic and Fermionic Gases, edited by K. Levin, A.L. Fetter and D. M. Stamper-Kurn, Concepts of condensed matter science(Elseiver, Amesterdam, 2012)

23. T. Kohler etal., Rev. Mod. Phys. 78, 4 (2006)

24. G. B. Partidge etal., Phys. Rev. Lett.(PRL) 97, 190407 (2006)

25. M. Zaccanti etal., Phys. Rev. A74, 041605 (2006)

26. M. M. Parish etal, Nature Phys. 3, 124 (2004)

27. T. Kohler etal., Rev. Mod. Phys. 78, 1311 (2006)

28. H. Pu etal., Phys. Rev.Lett.(PRL) 88,070408 (2002)

29. A. L. Fetter, Annals of Physics 70, 67 (1972)

30. E. Timmermans and R. Cote, Phys. Rev. Lett. (PRL) 80, 3419 (1998)

31. C. J. Pethick and H. Smith, “Bose-Einstein Condensation in dilute gases” (Cambridge University, press, Cambridge ,2002)

32. L. P. Pitaevskii and S. Stringari, “Bose-Einstein Condensation (Clarendon Press, Oxford, 2003)

33. I. Bloch etal., Nat. Phys. 8, 267 (2012)

34. A. E. S. Green , Phys. Rev. A26, 1759 (1982)

35. D. M. Stamper-Kurn etal, Phys. Rev. Lett. (PRL) 80, 2027 (1998)

36. T. Lufforrd etal., Phys. Rev. Lett. (PRL) 88, 173201 (2002)

37. M. O. Mewes etal., Phys. Rev. Lett. (PRL) 77, 416 (1996)

38. S. Giorgini etal., Phys. Rev. A54, R4633 (1996)

39. K. Dieckmann etal., Phys. Rev. Lett. (PRL) 89, 203201 (2002)

40. S. B. Papp etal., Phys. Rev. Lett. (PRL) 101, 040402 (2008)

41. K. Aikawa etal., New J. Phys. 11, 055035 (2009)

42. S. Kotochigova, etal., Phys. Rev. A82, 063421 (2010)

43. M. H. G. De Miranda etal., Nat. Phys. 7, 502 (2011)

44. Jee Park Woo etal., Phys. Rev. A85, 01602(R) (2012)

45. M. H. Divoret etal., Science 339, 1169 (2013)

46. N. Y. Yao etal., Phys. Rev A 113, 243002 (2014)

47. J. S. Douglas etal., Nat. Photon, 9, 326 (2015)

48. M. Gao etal., Phys. Rev. Lett. (PRL) 116, 205303 (2016)

49. L. V. Kulik etal., Nat. Commun. 7, 13499 (2016)

50. Peletminskii etal., J. Phys. B, At. Mol.and optical phys. 50, 14, 22 June (2017)