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## A THEORETICAL EVALUATION OF MOMENTUM DISTRIBUTION, BCS PAIR FUNCTION, BCS PAIR DISTRIBUTION FUNCTION, IN-COMPRESSIBILITY, EFFECTIVE MASS AND SOUND VELOCITY OF ATOMIC FERMI GAS NEAR THE UNITARITY LIMIT.

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### Abstract:

- Using the theoretical formalism of S. De Palo et al [Laser Physics, 15, 376 (2005)], we have evaluated momentum distribution  $n(k)$  of atomic Fermi gas near the unitarity limit as a function of  $k/k_F$  for

different values of  $g\sqrt{n}/E_F$  and also different values of  $T/T_F$ . Our theoretically evaluated results show that  $n(k)$  decrease as  $k/k_F$  for both fixed value of  $g\sqrt{n}/E_F$  and also for fixed value of  $T/T_F$ . This type of behaviour is an indication of normal Fermi gas character with large value of jump at  $k=k_F$ . This results also demonstrate that there is a BCS-like solutions for narrower well and momentum distribution

$n(k)$  is more pronounced with bosonic character. Our theoretically evaluated results for BCS-pair function  $\psi(r)$  as a function of  $(r/a_0)$  for different values of  $g\sqrt{n}/E_F$  show that node of wave function is shifted towards larger value of  $r$ . Our obtained results indicate that the size of the Cooper pair is related to the spatial extension of the pair function. Our theoretically evaluated results of BCS pair distribution

function  $g(r)$  as a function of  $[r/(r_s a_0)]$  for different values of  $T/T_F$  show that  $g(r)$  increase with  $[r/(r_s a_0)]$  for each value of  $T/T_F$ . The results also indicate that  $g(r)$  has a higher peak as  $T \rightarrow 0$ . But the peak is washed out as  $T$  approaches  $T_c$ . Using the theoretical formalism of G. Watanabe et al [arXiv:1301.3363v4(cond.mat.quant.gas) Oct 2013], we have evaluated ratio

$[k^{-1}/k^{-1}(s=0)]$ ,  $(m^*/m)$  and  $[c_s/c_s^{(0)}]$  all as a function of  $(\frac{E_F}{E_R})$  for different values of  $s$ , where  $E_F$  is Fermi energy,  $E_R$  is recoil energy and 's' is the lattice intensity in the dimensionless unit. Our theoretically evaluated results indicate that  $[k^{-1}/k^{-1}(s=0)]$ ,  $(m^*/m)$  and  $[c_s/c_s^{(0)}]$  all increase first, attain maximum value and then decrease as a

function of  $(\frac{E_F}{E_R})$  for all different values of  $s$ . These results were obtained for unitary Fermi gas on the optical lattice. These parameters play an important role in the formation of molecules induced by the lattice. Our theoretically obtained results are in good agreement with those of other theoretical workers. The obtained results have great significance in the crossover physics of BCS and BEC states. As the scattering length diverges at the unitarity limit the present study of BCS and BEC Fermi super fluid have been performed in a unified point of view.

**Keywords:** Momentum distribution function, BCS pair function, BCS pair distribution function, size of Cooper pair, optical lattice, in-compressibility, effective mass, ratio of sound velocities, lattice intensity in the dimensionless unit, recoil energy, Fermi energy



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## INTRODUCTION

In an earlier paper<sup>1</sup>, we have studied the behaviour across resonance in the unitary limit  $|a| \rightarrow \infty$ . As one knows that in the unitary limit, the thermodynamic properties are expected to be independent of the scattering length and the only available length scale is the inter particle distance<sup>2</sup>. However this argument fails in the case of resonance when the scattering phase shift significantly changes over the energy range of the Fermi energy<sup>3</sup>. The second length scale called effective range plays an important role. Using the theoretical formalism of S.De Palo et al.<sup>4</sup>, we have theoretically studied super fluidity of an atomic Fermi gas near the unitary limit. Using the theoretical formalism of Q. Chen<sup>5</sup>, we have theoretically studied super fluid transition temperature in an atomic Fermi gas on 3D isotropic lattice with an attractive on-site interaction in the unitary limit. Our theoretically evaluated results are in good agreement with the works of other theoretical workers<sup>6,7</sup>.

In this paper, using the theoretical formalism of S. De Palo<sup>4</sup>, we have analyzed the momentum distribution  $n(k)$  as a function of  $k/k_F$  for different values of  $g\sqrt{n}/E_F$ . These results illustrate the BCE-like solution for narrower wells which is accompanied by more pronounced bosonic character. We have also evaluated BCS pair function as a function of  $r/a_0$  in the unitarity limit with scattering length  $a=5000a_0$  and  $nr_0^3 = 2 \times 10^{-3}$  for different values of  $g\sqrt{n}/E_F$ . In this case the BCS pair functions  $\Psi(r)$  in the unitarity limit are associated for different values of  $g\sqrt{n}/E_F$ . The obtained results were compared with the solution of the two-body scattering problem<sup>7</sup>. These results also indicate that the average size of the Cooper pair is related to the spatial extension of the pair function. We have also evaluated BCS pair distribution function  $g(r)$  as a function of  $[r/(r_s a_0)]$  in the unitarity limit keeping  $a=5000a_0$  and  $nr_0^3 = 0.125$ . In this evaluation, we have kept the value of  $g\sqrt{n}/E_F$  fixed about 0.95 and evaluated  $g(r)$  as a function of  $[r/(r_s a_0)]$  for different values of  $(T/T_F)$ . These results show that the pair distribution function  $g(r)$  which is large at  $T=0$  is washed away as the  $T$  approaches  $T_F$ . Using the theoretical formalism of G Watanabe et al.<sup>8</sup>, we have evaluated incompressibility, effective mass and sound velocity of unitary Fermi gas. We have evaluated  $[k^{-1}/k^{-1}(s=0)]$  as a function of  $E_F/E_R$  for different values of  $s$ . Here  $s$  is the lattice intensity in the dimensionless

unit,  $E_F$  is the Fermi energy and  $E_R$  is the recoil energy. The evaluation is performed for different values of  $s$ ,  $s=1$ ,  $s=3$  and  $s=5$  respectively. Our theoretically obtained results indicate that  $[k^{-1} / k^{-1}(s=0)]$  take maximum value larger than unity in the intermediate region of  $E_F / E_R$ . We have also calculated effective mass ( $m^*/m$ ) as a function of  $E_F / E_R$  for different values of  $s$ . In other evaluation, we have evaluated sound velocity of the unitary Fermi gas in a lattice. We have evaluated  $[c_s / c_s^{(0)}]$  as a function of  $E_F / E_R$  for different values of  $s$ . Here,  $c_s$  is the sound velocity in the unitary Fermi gas and  $c_s^{(0)}$  is the sound velocity of the uniform system. Our theoretically obtained results indicate that  $[c_s / c_s^{(0)}]$  as the function of  $E_F / E_R$  decrease as the value of  $s$  increase. Our theoretically evaluated results are in good agreement with yje other theoretical workers<sup>9,10</sup>.

**MATERIALS AND METHODS**

One develops a model that is able to interpolate between two limits of a broad resonance. Here, the interaction is studied with the help of scattering length from low energy renormalization and of a narrow resonance in a broad Fermi sea. One models minimal interaction potential that is able to reproduce and independently tune the three relevant features of a Feshbach resonance. These are detuning and the width of the resonance and the back-ground scattering length  $a_{bg}$ . The thermodynamics properties of atomic gas in the unitary limit are investigated as function of resonance width by resorting to a mean-field solution of the BCS-like equations.

**Evaluation of momentum distribution function  $n(k)$ , BCS pair function  $\psi(r)$  and BCS pair distribution function**

One models the presence of Feshbach resonance by the interaction potential given by

$$\begin{aligned}
 V(r) &=V_0 \quad r<r_0 \\
 &=V_1 \quad r_0<r<r_1 \\
 &=0 \quad \text{otherwise} \quad (1)
 \end{aligned}$$

The potential is characterized by an attractive well depth  $V_0$  and width  $r_0$  and a barrier with height  $V_1$  and width  $r_1 - r_0 = r_w$ . This barrier –well model allows one to incorporate the essential

energy dependence of the scattering physics into a single channel scattering scheme. The well can support resonant states with a width dependent on the tunnelling through the potential barrier. Here, there are three distinct parameters like diluteness  $na^3$ , the interaction strength

$na^3$  and the width  $\Delta V$  of the resonance on the scale of the Fermi energy  $E_F = \frac{\hbar^2 k_F^2}{2m}$  with

$k_F = (3\pi^2 n)^{\frac{1}{3}}$ .  $\Delta V$  can be expressed in terms of the matrix element  $g$  for the coupling

between the closed and open channels as  $\Delta V = g\sqrt{n}$ ,  $n$  is the particle density.

One requires that the diluteness conditions  $na^3 \ll 1$  is always satisfied while staying within the

unitary limit  $na^3 \gg 1$ . One thus vary the width of the resonance by tuning the parameter  $\frac{\Delta V}{E_F}$ .

The parameters of the model potential can in principle be adjusted to reproduce the scattering properties of an atomic sample<sup>11</sup>. One chooses two sets of parameters that are suitable for the physical behaviour of the sample. One fixes the scattering length to a large and positive values  $a=5000a_0$  ( $a_0$  is Bohr radius). This is for staying on BEC side of the resonance under unitary-limited conditions. The other parameter is range  $r_0$  which is taken to be equal to  $r_0 = 500a_0$  such that  $na^3=0.002 \ll 1$ . Now, one solves the scattering problem for  $V(r)$  shown in equation (1) in the standard way. Then, one determines the two-body scattering functions  $\Psi(r)$  and the T-matrix<sup>12</sup>.

The T-matrix contains all the necessary information. The scattering length  $a$  is determined from its definition

$$\lim_{k \rightarrow 0} T(k) = \frac{4\pi\hbar^2 a}{m} \quad (2)$$

The usual relation is

$$T(k) = 2\pi\hbar^2 i[S(K) - 1] / mk \quad (3)$$

The Feshbach form for the S-matrix in the one resonance parameterization is given by

$$S(k) = e^{-2ika_{bg}} \left[ 1 - \frac{2ik|g|^2}{-\frac{4\pi\hbar^2}{m} \left( v - \frac{\hbar^2 k^2}{m} \right) + ik|g|^2} \right] \quad (4)$$

The parameter  $g$  is defined in terms of the effective range<sup>13</sup>

$$R_{\text{eff}} = -\left( \frac{4\pi\hbar^2}{m} \right) \left[ \frac{d^2 T(K)^{-1}}{dk^2} \right]_{k=0} \quad (5)$$

$R_{\text{eff}}$  is always negative close to resonance. One obtains

$$|g|^2 = -\frac{8\pi\hbar^2}{mR_{\text{eff}}} \quad (6)$$

One has removed the background  $a_{bg} = 0$

Now, one finds the equilibrium state of the Fermi gas interacting via the non-local potential interaction modeled by  $V(r)$ . The ground state is determined within the usual variational BCS scheme that utilizes the wave functions<sup>14</sup>

$$|\Phi_0\rangle = \prod_k (u_k + v_k a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger) |0\rangle \quad (7)$$

where  $a_{k\sigma}^\dagger$  is the creation operator for the electrons in spin  $\sigma$ .  $|\Phi_0\rangle$  is normalized giving the condition  $|u_k|^2 + |v_k|^2 = 1$ . The expected value of the ground state energy at  $T=0$  is given by

$$E_0 = \sum_k 2\varepsilon_k |v_k|^2 + \sum_{kk'} V_{kk'} u_k v_k^* u_{k'} v_{k'}^* + \sum_{kk'} V_{kk'} u_k u_k^* v_{k'} v_{k'}^* \quad (8)$$

Now, one solves the above equation by converting the summations into one-dimensional integral

$$\sum_k V_{kk'} F(k \rightarrow ') = \frac{1}{(2\pi)^3} \int dq q^2 V(k, q) F(q) \quad (8)$$

$V(k,q)$  is determined from the three-dimensional Fourier transform

$$V(k,q) = \frac{2\pi}{kq} (V_0 + V_1) \left[ \frac{\sin r_0 |k+q|}{|k+q|} - \frac{\sin r_0 |k-q|}{|k-q|} \right] \\ - V_1 \left[ \frac{\sin r_1 |k+q|}{|k+q|} - \frac{\sin r_1 |k-q|}{|k-q|} \right] \quad (9)$$

The BCS solution results from the minimization of the free energy with respect to the variational parameter  $u_k$  and  $v_k$  and chemical potential  $\mu$  fixed by the constraint of particle-density conservation

$$f = \langle \Phi_0 | H | \Phi_0 \rangle - \mu \langle \Phi_0 | N | \Phi_0 \rangle \quad (10)$$

The two resulting self-consistent equations for the isotropic super fluid gap and the particle density are given by

$$\Delta(k) = \frac{1}{(2\pi)^3} \int dq q^2 V(k,q) \frac{\Delta(q)}{2E(q)} \quad (11a)$$

$$n = \frac{1}{(2\pi)^3} \int dk \rightarrow n_k \quad (11b)$$

$$n_k = 1 - \frac{\xi(k \rightarrow)}{E(k \rightarrow)} \quad (11c)$$

In equations 11(a) to 11(c) the excitation energy  $E(k)$  is expressed as

$$E(k) = \sqrt{\Delta(k)^2 + \xi(k)^2} \quad (12)$$

Here,  $\Delta(k)$  is gap function and  $\xi(k \rightarrow)$  is the single-particle energy and is given by

$$\xi(\mathbf{k} \rightarrow) = \varepsilon_{\mathbf{k}} - \mu + \frac{1}{2} V_{\mathbf{k}=\mathbf{q}=0} n - \sum_{\mathbf{k}} V_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}'} \quad (13)$$

In this equation, the terms second and third contains the Hartee-Fock corrections to the single particle energy. Equations 11(a) and 11(b) are solved self-consistently to find  $\Delta(\mathbf{k})$  at each  $\mathbf{k}$ -vector and  $\mu$ . The Bogoliubov parameters  $u_{\mathbf{k}}$  and  $v_{\mathbf{k}}$  were evaluated and the values are given by

$$u_{\mathbf{k}} = \Delta(\mathbf{k}) \left[ \frac{1}{2} \left( 1 + \frac{\xi(\mathbf{k})}{E(\mathbf{k})} \right) \right]^{\frac{1}{2}} \quad (14(a))$$

$$v_{\mathbf{k}} = \left[ \frac{1}{2} \left( 1 - \frac{\xi(\mathbf{k})}{E(\mathbf{k})} \right) \right]^{\frac{1}{2}} \quad (14(b))$$

At finite temperature, the equations are modified to include Fermi function  $f(\mathbf{k})$  which is given by

$$f(\mathbf{k} \rightarrow) = [\exp((\xi(\mathbf{k} \rightarrow) / K_{\beta} T)]^{-1} \quad (15(a))$$

$$\Delta(\mathbf{k}) = \frac{1}{(2\pi)^3} \int d\mathbf{q} q^2 V(\mathbf{k}, \mathbf{q}) \frac{\Delta(\mathbf{q})}{E(\mathbf{q})} (1 - 2f(\mathbf{q})) \quad (15(b))$$

$$n = \frac{1}{(2\pi)^3} \int d\mathbf{k} \rightarrow n_{\mathbf{k}} \quad (15(c))$$

$$n_{\mathbf{k}} = 2v_{\mathbf{k}}^2 (1 - f(\mathbf{k})) + 2u_{\mathbf{k}}^2 f(\mathbf{k}) \quad (15(d))$$

The single-particle excitation energy  $\xi(\mathbf{k} \rightarrow)$  is modified accordingly by substituting the new  $n_{\mathbf{k}}$  from equation 15(c) and 15(d).

The BCS pair function is defined as

$$\psi(\mathbf{r}) = \int d\mathbf{r} \rightarrow (v_{\mathbf{k} \rightarrow} / u_{\mathbf{k} \rightarrow}) \exp(i\mathbf{k} \rightarrow \mathbf{g} \rightarrow) \quad (16)$$

**Evaluation of in-compressibility, effective mass and sound velocity of unitary Fermi gas in a lattice**

One obtains  $k^{-1}$  of unitary Fermi gas<sup>15</sup>

$$k^{-1} ; \frac{2}{3}(1+\beta)E_F [1 + \frac{1}{32}(1+\beta)^{-2}(\frac{sE_R}{E_F})^2] + o[(\frac{sE_R}{E_F})^4] \dots \quad (17)$$

Here  $E_R = \frac{\hbar^2 q_B^2}{2m}$  is the recoil energy,  $E_F = \frac{\hbar^2 k_F^2}{2m}$  is the Fermi energy,  $q_B = \pi/d$  is

Bragg wave vector,  $s$  is the lattice intensity in the dimensionless unit and  $\beta$  is an universal parameter which is negative and its absolute value is of order of unity, accounting for attractive inter atomic interactions<sup>16,17</sup>

The expression for  $m^*$  of unitary Fermi gas is given by

$$\frac{m^*}{m} ; 1 + \frac{9}{32}(1+\beta)^{-2}(\frac{sE_R}{E_F})^2 + o[(\frac{sE_R}{E_F})^4] \quad (18)$$

The sound velocity of the unitary Fermi gas is given by

$$c_s = \sqrt{\frac{k^{-1}}{m^*}} \quad (19)$$

One obtains the expression of the sound velocity in the weak lattice limit<sup>18</sup>

$$c_s^2 = c_s^{(0)2} [1 - \frac{1}{4}(1+\beta)^{-2}(\frac{sE_R}{E_F})^2 + o[(\frac{sE_R}{E_F})^4]] \quad (20)$$

$$c_s^{(0)} = [(\frac{2}{3}(1+\beta)E_F / m)^2]^{\frac{1}{2}} \quad (21)$$

This is the sound velocity of the uniform system



**RESULTS AND DISCUSSION;**

Using the theoretical formalism of S.De Palo et al<sup>4</sup>, we have studied super fluidity of an atomic Fermi gas near the unitary limit. The studies show that the thermodynamic properties of an atomic gas near the unitary limit are not universal as long as the resonance is narrow on the scale of Fermi energy. This formalism uses a mean-field BCS-like approach with a model potential that is able to reproduce the main character of Feshbach resonance. The formalism also gives a complete theoretical description of the crossover from BEC of composite bosons to BCS-type of Cooper pairs. It was also observed that the thermodynamic properties of the super fluid with large and positive scattering length on BEC side of resonance are affected by the energy dependence of the phase shift. The narrow resonance favours the occurrence of the strong coupling super fluidity. **In table T1**, we have presented an evaluated results of momentum distribution  $n(k)$  as a function of  $k/k_F$  of atomic Fermi gas in the unitarity limit.

We have taken the value of scattering length  $a=5000a_0$  and  $nr_0^3 = 0.125$  and evaluated  $n(k)$  for different values of  $g\sqrt{n}/E_F$ . We have taken  $g\sqrt{n}/E_F=1.0, 2.0, 5.0$  and  $10.0$ . Our theoretically evaluated results show that  $n(k)$  decrease with  $k/k_F$  for all the values of  $g\sqrt{n}/E_F$ . The decrease is more pronounced for large value of  $g\sqrt{n}/E_F$ . This type of behaviour is an indication of normal Fermi gas character with large value of jump at  $k=k_F$ . **In table T2**, we repeated the calculation of  $n(k)$  as a function of  $k/k_F$  for different values of  $T/T_F$

keeping the value of  $a=5000a_0$  and  $nr_0^3 = 0.125$  and  $g\sqrt{n}/E_F=0.95$ . Our theoretically obtained results show that here also  $n(k)$  decrease as  $k/k_F$  for each value of  $T/T_F$ . The decrease is more pronounced for large value of  $T/T_F$ . These results demonstrate that BCS-like solution exist for narrower wells. Momentum distribution function is more pronounced with bosonic character.

**In table T3**, we have shown an evaluated results of BCS pair function  $\Psi(r)$  as a function of  $(r/a_0)$  for different values of  $g\sqrt{n}/E_F$ . The evaluation is performed for  $g\sqrt{n}/E_F=0.8, 1.6, 2.5$  and  $10.5$ . Our theoretical results show that  $\Psi(r)$  decrease with  $(r/a_0)$  for each value of  $g\sqrt{n}/E_F$ . These results also indicate that for broad resonances the node of the wave function is shifted towards larger value of  $r$ . Results also give an idea about the size of the Cooper pair. The size of the Cooper pair is related with the spatial extension of the pair wave function. Cooper pair is seen to become smaller when the resonance

shrinks. In table T4, we have presented an evaluated results of BCS=pair distribution function  $g(r)$  as a function of  $[r / (r_s a_0)]$  for different values of  $T/T_F$ . In this calculation, we have kept  $a=5000a_0$ ,  $nr_0^3 = 0.125$  and  $g\sqrt{n} / E_F = 1.0$ . Our theoretical obtained results show that  $g(r)$  increase with  $[r / (r_s a_0)]$  for each value of  $T/T_F$ . The results show that  $g(r)$  has a higher peak as  $T \rightarrow 0$ . But the peak is washed out as  $T \rightarrow T_F$ . Using the theoretical formalism of G. Watanabe<sup>8</sup>, we have evaluated  $[k^{-1} / k^{-1}(s = 0)]$  as a function of  $(E_F / E_R)$  for different values of  $s$ . The results are shown in table T5 for unitary Fermi gas in an optical lattice. Our theoretically obtained results show that the ratio  $[k^{-1} / k^{-1}(s = 0)]$  first increase with  $(E_F / E_R)$  attains maximum value and then decrease. The trend is same for all values of  $s$  taken as  $s=1, 2, 3, 4$  and  $6$ . However, the value of the ratio is higher for  $s=6$  and lower for  $s=1$ . In table T6, we have shown the evaluated results of effective mass  $(m^* / m)$  as a function of  $(E_F / E_R)$  for same value of  $s$ . Here, one observed that  $(m^* / m)$  increase with  $(E_F / E_R)$  attain maximum value and then decrease for each value of  $s$  from  $1$  to  $6$ . In this case also  $(m^* / m)$  is large for  $s=6$  and small for  $s=1$ . In table T7, we have presented an evaluated results of sound velocities  $[c_s / c_s^{(0)}]$  as a function of  $(E_F / E_R)$  for value of  $s$  from  $1$  to  $6$ . Our theoretical results show that here also sound velocities  $[c_s / c_s^{(0)}]$  increase with  $(E_F / E_R)$  for each value of  $s$ . But in this case its value is large for  $s=1$  and small for  $s=6$ . Here  $c_s^{(0)}$  is the sound velocity for uniform system. All these three results of the ratio of in-compressibility, effective mass and sound velocities for unitary Fermi gas in an optical potential play a crucial role in the formation of molecules induced by lattice. There is some recent calculations<sup>19-30</sup> in this field which reveals the similar behaviour.

## CONCLUSIONS

From the above theoretical investigations and analysis, we have come across the following conclusions:

(1) We obtained momentum distribution  $n(k)$  as a function of  $k/k_F$  for different values of  $g\sqrt{n} / E_F$  for atomic Fermi gas in an unitarity limit. Our theoretically obtained results show

that  $n(k)$  decrease with  $k/k_F$  for each value of  $g\sqrt{n}/E_F$ . The decrease is more pronounced for large value of  $g\sqrt{n}/E_F$ . This type of behaviour is an indication of normal Fermi gas character with large value of jump at  $k=k_F$ .

(2) We have repeated the calculation of  $n(k)$  as  $k/k_F$  for different values of  $T/T_F$ . Here also, we observed that  $n(k)$  decrease with  $k/k_F$  for each value of  $T/T_F$  and decrease is more pronounced for large value of  $T/T_F$ . These results demonstrate that BCS-like solution for narrower wells.  $n(k)$  is more pronounced with the bosonic character.

(3) Our theoretically obtained results of BCS pair function as a function of  $(r/a_0)$  for different values of  $g\sqrt{n}/E_F$  show that for broad resonance the node of the wave function is shifted towards large value of  $r$ . These results give an idea about the size of the Cooper pair. The size of the Cooper pair is related with the spatial extension of the pair wave function. Cooper pair is seen to become smaller when the resonance shrinks

(4) Our theoretically evaluated results of BCS pair distribution function  $g(r)$  as a function of  $[r/(r_s a_0)]$  for different values of  $T/T_F$  indicate that  $g(r)$  increase with  $[r/(r_s a_0)]$  for each value of  $T/T_F$ . For large value of  $[r/(r_s a_0)]$ ,  $g(r)$  value coincides for each value of  $T/T_F$ . At  $[r/(r_s a_0)] = 2.9$ , all  $g(r)$  value merges. These results show that  $g(r)$  has a higher peak as  $T \rightarrow 0$ . But the peaks are washed out as  $T \rightarrow T_F$ .

(5) We have evaluated the ratio of in-compressibility  $[k^{-1}/k^{-1}(s=0)]$ , ratio of effective mass  $(m^*/m)$  and sound velocities  $[c_s/c_s^{(0)}]$  all as a function of  $(E_F/E_R)$  for different values of  $s$  from  $s=1$  to  $s=4$ . Our obtained theoretical results show that  $[k^{-1}/k^{-1}(s=0)]$ ,  $(m^*/m)$  first increase with  $(E_F/E_R)$  attain maximum value and then decrease for each value of  $s$ . However, ratio of sound velocities  $[c_s/c_s^{(0)}]$  increases with  $(E_F/E_R)$  for all values of  $s$ . These results were obtained for unitary Fermi gas in the optical lattice. The formation of the molecules induced by lattice is pronounced by the behaviour of  $k^{-1}$ ,  $m^*$  and  $c_s$ .

(6)The entire results of different parameters investigated in the unitary limit gives an idea about the behaviour of the gas in the context of scattering length and Feshbach resonance. Since the scattering length diverges in this limit the obtained results will give good insight in order to formulate good and reliable theory for BCS-BEC crossover.

**Table T1: An evaluated result of momentum distribution  $n(k)$  as a function of  $k/k_F$  of the Fermi gas in the unitarity limit with scattering length  $a = 5000a_0$  and  $nr^3_0 = 0.125$  with different values of  $g\sqrt{n} / E_F$**

<----- Momentum distribution $n(k)$ ----->				
$k/k_F$	$g\sqrt{n} / E_{F=1.0}$	$g\sqrt{n} / E_{F=2.0}$	$g\sqrt{n} / E_{F=5.0}$	$g\sqrt{n} / E_{F=10.0}$
0.00	0.078	0.384	0.897	1.004
0.50	0.069	0.376	0.806	0.105
1.00	0.057	0.365	0.755	0.053
1.50	0.052	0.320	0.702	0.008
2.00	0.048	0.284	0.643	0.004
2.50	0.037	0.255	0.606	0.002
3.00	0.032	0.207	0.554	0.001
3.50	0.026	0.184	0.500	0.000
4.00	0.022	0.137	0.422	0.000
4.50	0.018	0.109	0.365	0.000
5.00	0.016	0.086	0.306	0.000
5.50	0.011	0.062	0.257	0.000
6.00	0.009	0.047	0.185	0.000

Table T2: An evaluated results of momentum distribution  $n(k)$  as a function of  $k/k_F$  for different values of  $T/T_F$ . The evaluation is performed by taking  $T/T_F = 0.05, 0.20, 0.50$  and  $0.80$ .

We have kept  $a=5000a_0$ ,  $nr_0^3=0.125$  and  $g\sqrt{n} / E_F = 0.95$

$k/k_F$	<---- Momentum distribution $n(k)$ ----->			
	$T/T_F=0.05$	$T/T_F=0.20$	$T/T_F=0.50$	$T/T_F=0.80$
0.00	0.954	0.963	0.998	1.000
0.20	0.923	0.924	0.957	0.997
0.40	0.908	0.899	0.902	0.576
0.50	0.886	0.864	0.853	0.329
0.70	0.853	0.783	0.729	0.184
1.00	0.802	0.720	0.658	0.105
1.15	0.789	0.629	0.592	0.062
1.20	0.686	0.548	0.485	0.037
1.25	0.534	0.446	0.392	0.006
1.30	0.475	0.365	0.314	0.000
1.40	0.389	0.269	0.243	0.000
1.50	0.305	0.212	0.187	0.000
1.60	0.276	0.176	0.132	0.000
1.70	0.228	0.108	0.096	0.000
1.80	0.187	0.706	0.058	0.000

Table T3: An evaluated result of BCS pair function  $\psi(r)$  as a function of  $r/a_0$  of the Fermi gas in the unitarity limit with  $a=5000a_0$  and  $nr^3_0 = 0.125$ . The evaluation is performed for different values of  $g\sqrt{n}/E_F = 0.8, 1.6, 2.5$  and  $11.5$ .

$r/a_0$	←----- BCS Pair function $\psi(r)$ -----→			
	$g\sqrt{n}/E_{F=0.8}$	$g\sqrt{n}/E_{F=1.6}$	$g\sqrt{n}/E_{F=2.5}$	$g\sqrt{n}/E_{F=11.5}$
100	1.00	1.262	1.304	1.027
500	0.907	1.105	0.846	1.184
1000	0.826	0.923	0.532	0.976
1500	0.743	0.876	0.476	0.843
2000	0.407	0.439	0.386	0.639
3000	0.382	0.347	0.320	0.542
4000	0.307	0.242	0.296	0.478
5000	0.276	0.207	0.253	0.389
6000	0.265	0.168	0.214	0.327
7000	0.240	0.143	0.185	0.276
8000	0.228	0.128	0.169	0.218
9000	0.204	0.107	0.122	0.179
$10^4$	0.195	0.085	0.108	0.122
$10^5$	0.173	0.069	0.082	0.064

**Table T4: An evaluated result of BCS pair distribution function  $g(r)$  as a function of  $[r / (r_s a_0)]$  of Fermi gas in the unitarity limit for different values of  $T/T_F$ , we have kept other parameters scattering length  $a=5000a_0$ ,  $nr_0^3 = 0.125$  and  $g\sqrt{n} / E_F = 1.0$**

$[r / (r_s a_0)]$	<----- BCS Pair distribution function $g(r)$ ----->				
	$T / T_F = 0.02$	$T / T_F = 0.5$	$T / T_F = 0.8$	$T / T_F = 0.9$	$T / T_F = 0.95$
0.00	0.157	0.387	0.796	0.837	0.857
0.50	0.227	0.433	0.882	0.889	0.908
1.00	0.348	0.587	0.845	0.907	0.954
1.50	0.469	0.659	0.924	0.928	0.996
2.00	0.568	0.796	0.987	0.956	1.034
2.50	0.686	0.843	0.995	1.043	1.057
2.60	0.725	0.927	1.034	1.057	1.069
2.70	0.879	0.974	1.056	1.069	1.075
2.80	0.893	0.983	1.065	1.084	1.083
2.90	0.948	0.992	1.079	1.095	1.095
3.00	0.967	1.026	1.115	1.105	1.106
3.20	1.006	1.049	1.127	1.122	1.117
3.40	1.104	1.056	1.143	1.138	1.132
3.50	1.132	1.104	1.156	1.147	1.145

Table T5: An evaluated result of  $[k^{-1} / k^{-1}(s = 0)]$  as a function of  $E_F / E_R$  for different values of  $s$  of unitary Fermi gas in an optical lattice

$E_F / E_R$	$\leftarrow [k^{-1} / k^{-1}(s = 0)] \rightarrow$				
	S=1	S=2	S=3	S=4	S=5
0.00	0.204	0.427	0.897	0.915	0.786
0.50	0.252	0.684	0.923	0.934	0.849
1.00	0.386	0.889	1.058	0.954	0.986
1.20	0.469	1.032	1.097	1.058	1.066
1.40	0.832	1.088	1.144	1.103	1.118
1.50	1.006	1.124	1.132	1.157	1.167
2.00	1.010	1.106	1.118	1.143	1.152
2.20	1.024	1.092	1.107	1.136	1.144
2.40	1.038	1.076	1.095	1.122	1.135
2.50	1.047	1.053	1.076	1.108	1.122
3.20	1.053	1.045	1.055	1.095	1.108
3.50	1.067	1.036	1.048	1.086	1.097
4.00	1.078	1.027	1.037	1.073	1.085
4.20	1.084	1.022	1.029	1.066	1.077
4.50	1.092	1.017	1.020	1.058	1.066
5.00	1.100	1.008	1.016	1.043	1.059
6.00	1.124	0.998	1.008	1.022	1.045



Table T6: An evaluated result of effective mass ( $m^* / m$ ) as a function of  $E_F / E_R$  for different values of  $s$  of unitary Fermi gas in lattice

$E_F / E_R$	< ---- Effective mass ( $m^* / m$ ) ---->				
	S=1	S=2	S=3	S=4	S=6
0.00	1.148	2.456	3.489	6.827	9.586
0.50	1.187	2.508	4.566	7.428	10.227
1.00	1.258	2.559	5.186	8.146	11.438
1.20	1.306	2.607	5.294	8.923	12.532
1.40	1.279	2.584	5.377	9.546	13.228
1.50	1.262	2.526	5.466	10.358	12.087
2.00	1.243	2.488	5.527	9.243	11.576
2.20	1.207	2.429	5.503	8.107	10.223
2.40	1.188	2.386	5.488	7.923	9.862
2.50	1.165	2.324	5.427	7.248	8.648
3.00	1.155	2.287	5.408	6.874	7.229
3.50	1.147	2.246	5.384	5.225	6.437
4.00	1.138	2.220	5.345	4.187	5.884
4.20	1.126	2.186	5.302	3.998	4.556
4.50	1.115	2.167	5.298	2.677	3.873
5.00	1.103	2.104	5.277	2.148	2.947
6.00	1.089	2.089	5.206	2.105	2.228

Table T7: An evaluated result of ratio of sound velocity  $\left[\frac{c_s}{c_s^{(0)}}\right]$  as a function of  $E_F / E_R$  for different values of  $s$  for unitary Fermi gas in a lattice,  $c_s^{(0)}$  is the sound velocity of the uniform system.

$E_F / E_R$	$\left[\frac{c_s}{c_s^{(0)}}\right]$				
	S=1	S=2	S=3	S=4	S=6
0.00	0.827	0.745	0.702	0.684	0.522
0.50	0.839	0.753	0.716	0.693	0.536
1.00	0.845	0.768	0.722	0.705	0.555
1.10	0.856	0.775	0.735	0.717	0.564
1.20	0.867	0.789	0.742	0.723	0.578
1.30	0.884	0.797	0.756	0.735	0.585
1.40	0.897	0.802	0.768	0.744	0.598
1.50	0.902	0.816	0.772	0.756	0.609
2.00	0.924	0.833	0.784	0.768	0.616
2.20	0.948	0.845	0.795	0.772	0.627
2.40	0.953	0.855	0.806	0.785	0.633
2.50	0.966	0.867	0.812	0.796	0.643
3.00	0.975	0.893	0.827	0.804	0.655
4.00	0.984	0.912	0.834	0.815	0.662
5.00	0.997	0.947	0.845	0.822	0.674

6.00	1.002	0.964	0.862	0.834	0.689
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